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INSTITUTE OF NUCLEAR & PARTICLE PHYSICS

Ab initio Leading-Order Effective Interactions for Elastic Scattering of Nucleons from Light Nuclei

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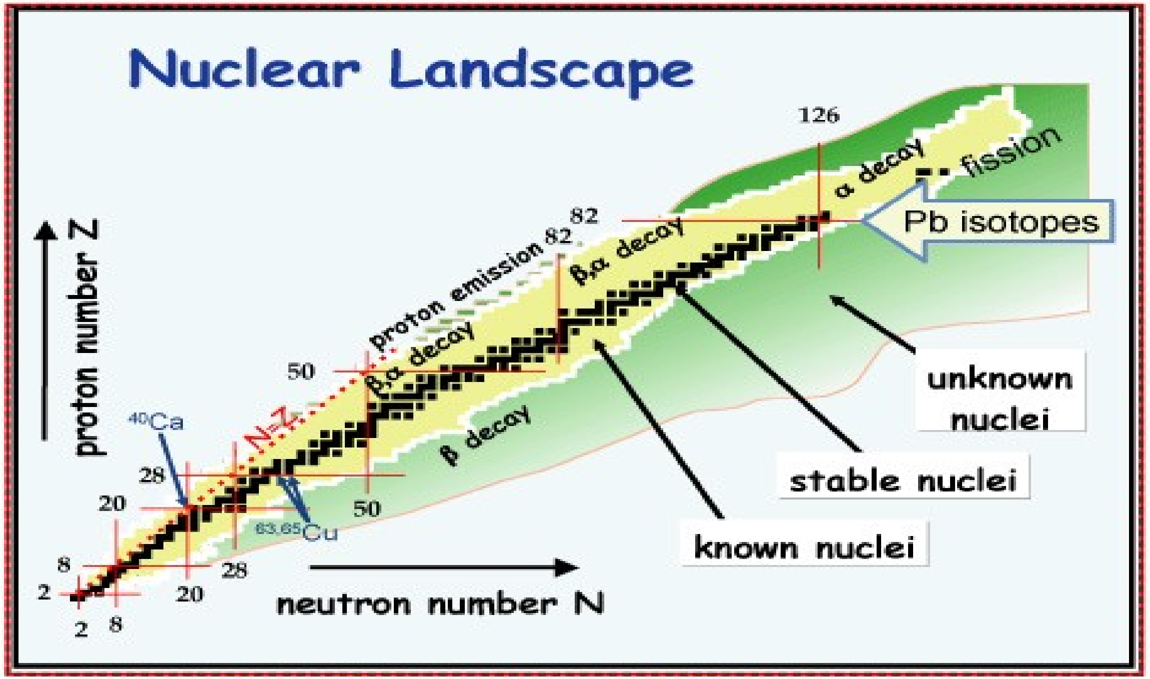
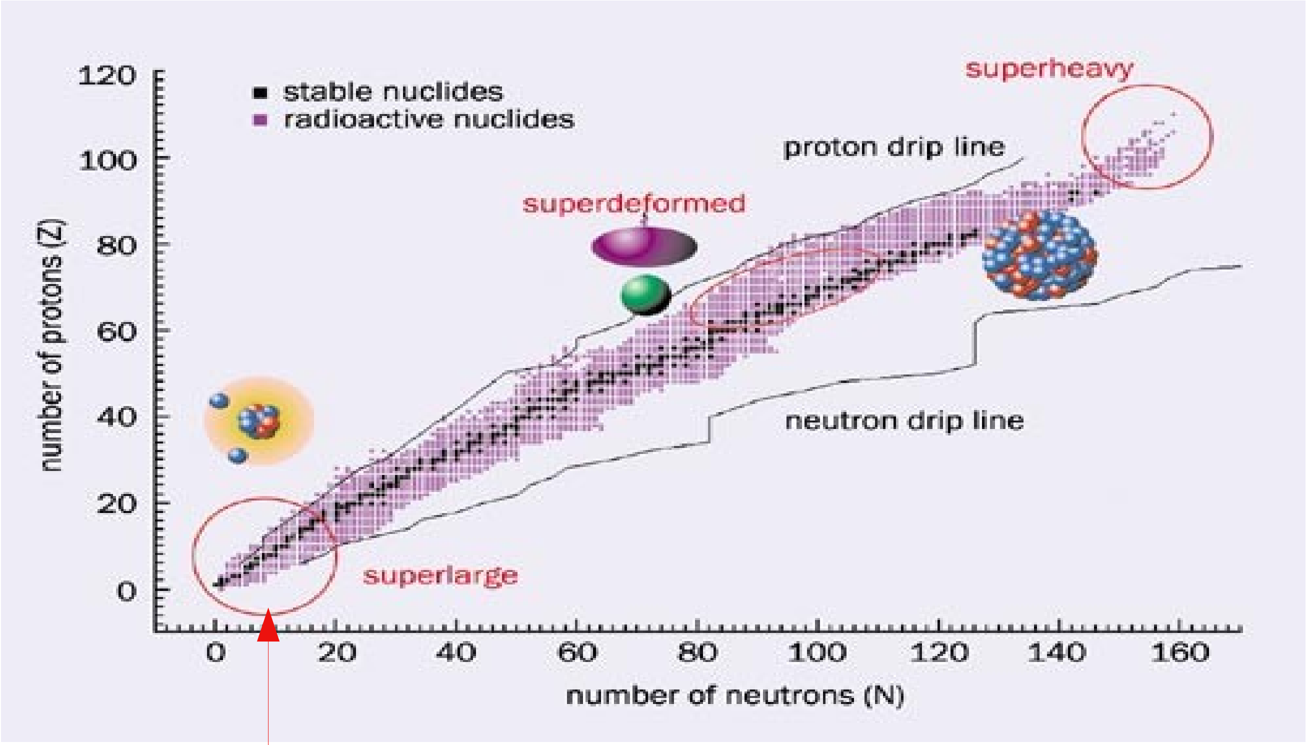
Entering the World of Exotic Nuclei



- **Exotic nuclei can be found in the crust of neutron stars**
- Extend our knowledge of the **nuclear force**
- Check the limits of validity of **structure** models
- **Is there life beyond the dripline**

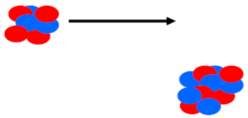
Exotic nuclei exhibit new phenomena



Regime of no-core shell-model (NCSM)

How do we learn about nuclei: Reactions

Elastic Scattering:



Traditionally used to extract optical potentials, rms radii, density distributions

Eur. Phys. J. A **15**, 27–33 (2002)
DOI 10.1140/epja/i2001-10219-7

THE EUROPEAN PHYSICAL JOURNAL A

Nuclear-matter distributions of halo nuclei from elastic proton scattering in inverse kinematics

P. Egelhof[1,a], G.D. Alkhazov[2], M.N. Andronenko[2], A. Bauchet[1], A.V. Dobrovolsky[1,2], S. Fritz[1], G.E. Gavrilov[2], H. Geissel[1], C. Gross[1], A.V. Khanzadeev[2], G.A. Korolev[2], G. Kraus[1], A.A. Lobodenko[2], G. Münzenberg[1], M. Mutterer[3], S.R. Neumaier[1], T. Schäfer[1], C. Scheidenberger[1], D.M. Seliverstov[2], N.A. Timofeev[2], A.A. Vorobyov[2], and V.I. Yatsoura[2]

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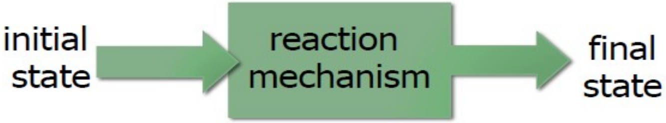
Matter distributions for ^{6,8}He and ^{6,8,9,11}Li measured

Exotic Nuclei are usually short lived:

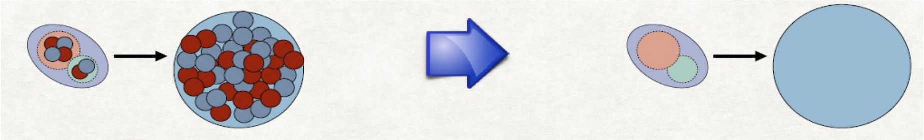
Have to be studied with reactions in inverse kinematics

e.g. direct reaction:



Challenge:

- **In the continuum, theory can only solve the few-body problem exactly.**



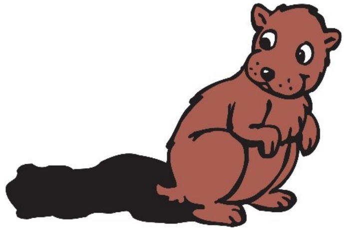
Many-body problem

Few-body problem

Example (d,p) Reactions:

Reduce Many-Body to Few-Body Problem

Solve few-body problem



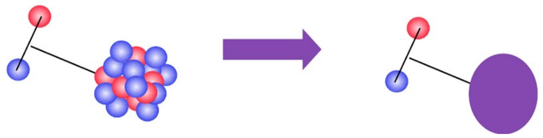
Challenges & Opportunities

Hamiltonian for effective few-body poblem:

$$H = H_0 + V_{np} + V_{nA} + V_{pA}$$

- **Nucleon-nucleon interaction believed to be well known:**
  today: chiral interactions
- **Effective proton (neutron) nucleus interactions:**
  - purely phenomenological optical potentials fitted to data
  - optical potentials with theoretical guidance
  - **microscopic optical potentials (1990s)**
- **Goal: *Ab initio* effective interactions**

Isolate relevant degrees of freedom

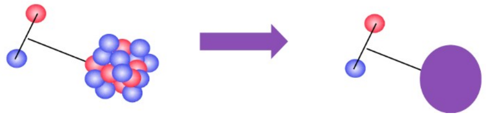


Formally: separate Hilbert space into **P** and **Q** space, and calculate in **P** space

Projection on **P** space requires introducing **effective interactions** between the degrees of freedom that are treated explicitly
(Feshbach, Annals Phys. 5 (1958) 357-390)

Effective Interactions: non-local and energy dependent

Isolate relevant degrees of freedom



Formally: separate Hilbert space into P and Q space, and calculate in P space

Projection on P space requires introducing **effective interactions** between the degrees of freedom that are treated explicitly
(Feshbach, Annals Phys. 5 (1958) 357-390)

Effective Interactions: non-local and energy dependent

Often used:

Phenomenological optical potentials

Either fitted to a large global data set OR to a restricted data set

(energy dependent, mostly local)

Most general form of optical potential

- Σ_i [$V_{A,Z,N,E}(r) + i\, W_{A,Z,N,E}(r)$] $\text{Operator}_{(i)}$
- **Functions are of Woods-Saxon type**

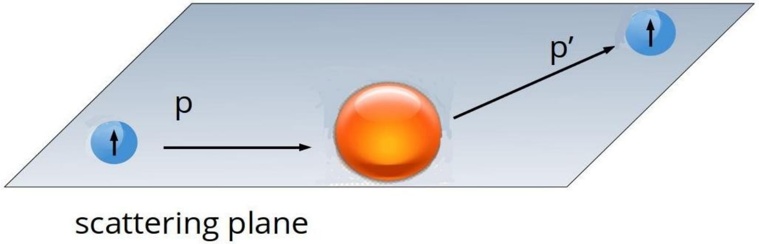
Have central and spin orbit term

Fit cross sections, angular distributions polarizations, for a set of nuclei (lightest usually ^{12}C).

No connection to microscopic theory

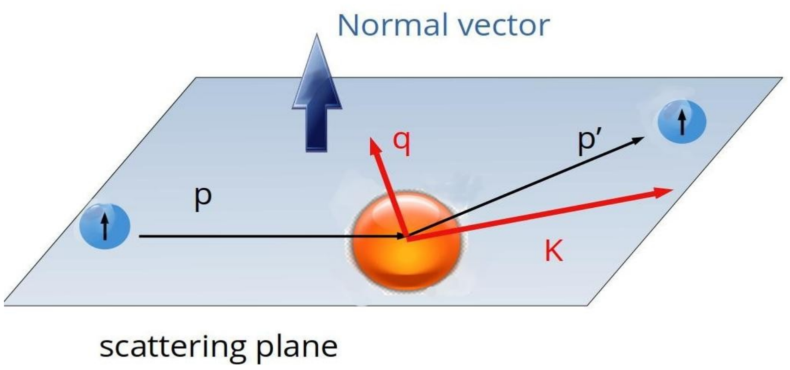
Dispersive optical models have some connection to structure

Phenomenological optical potentials for proton elastic scattering from 0^+ nuclei



- **Nucleus seen as absorptive sphere**
- **→ complex potential, local and energy dependent**
- **Nucleus has no spin**
- **Projectile proton (neutron) has spin = ½**
- **Scattering formalism: spin-1/2 on spin-0**
  - Differential cross section $d\sigma/d\Omega$
  - Analyzing power Ay, Spin Rotation Parameter Q

Proton elastic scattering from 0^+ nuclei



Variables:

In scattering plane:

Momentum transfer **q = p' - p**
Average momentum **K = ½ (p'+p)**

Normal to scattering plane: $\hat{n} = \hat{K} \times \hat{q}$

Spin-Orbit force in momentum space given by operator $\sigma \cdot \hat{n}$

Today: huge progress in *ab initio* structure calculations

Goal: effective interaction from *ab initio* methods

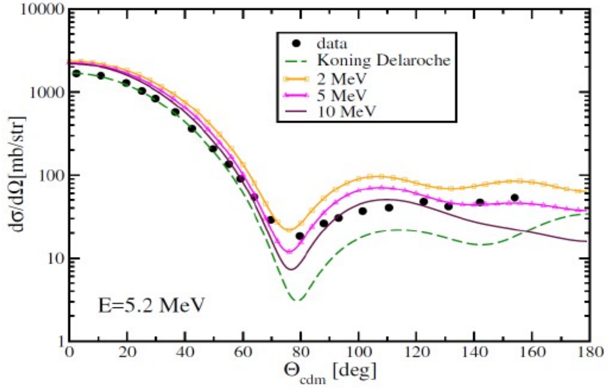
Start from many-body Hamiltonian with 2 and 3 body forces

Theoretical foundations laid by Feshbach and Watson in the 1950s

Feshbach:

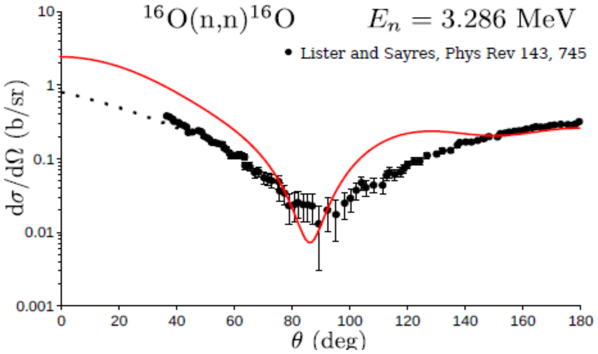
► effective nA interaction via Green's function from solution of many body problem using basis function expansion, e.g. SCGF, CCGF (current truncation to singles and doubles)

^{40}Ca(n,n)^{40}Ca



Rotureau, Danielewicz, Hagen, Jansen, Nunes
arXiv: 1808.04535 and PRC 95, 024315 (2017)

energy
~ 10 MeV



Idini, Barbieri, Navratil
J.Phys.Conf. 981. 012005 (2018)
Acta Phys. Polon. B48, 273 (2017)

Goal: effective interaction from *ab initio* methods

Start from many-body Hamiltonian with 2 (and 3) body forces

Theoretical foundations laid by Feshbach and Watson in the 1950s

Feshbach:

 effective nA interaction via Green's function from solution of many body problem using basis function expansion, e.g. SCGF, CCGF (current truncation to singles and doubles)

energy ~ 10 MeV

Watson:

► **Multiple scattering expansion, e.g. spectator expansion**

(current truncation to two active particles)

Spectator Expansion:

Siciliano, Thaler (1977)

Picklesimer, Thaler (1981)

Expansion in:

- **particles active in the reaction**
- **antisymmetrized in active particles**

Intended for "fast reaction", i.e. ≥ 100 MeV

Elastic Scattering (Watson approach)

- In- and Out-States have the target in ground state Φ_0
- Projector on ground state $\mathbf{P} = |\Phi_0\rangle\langle\Phi_0|$
- With **1=P+Q** and $[\mathbf{P}, \mathbf{G_0}]=0$
- For elastic scattering one needs: $\mathbf{P\,T\,P = P\,U\,P + P\,U\,P\,G_0(E)\,P\,T\,P}$
- $$\boxed{\begin{aligned} &\mathbf{T = U + U\,G_0(E)\,P\,T} \\ &\mathbf{U = V + V\,G_0(E)\,Q\,U} \end{aligned}}$$

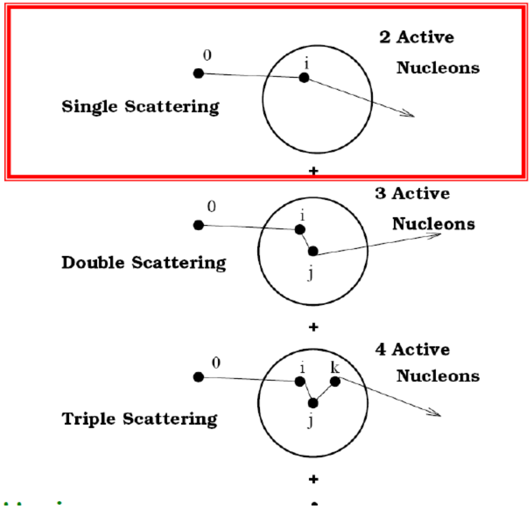
← **effective (optical) potential**

Exact expression

Spectator Expansion of U :

1st order: single scattering: $U^{(1)} \approx \sum \tau_{0i}$

Chinn, Elster, Thaler, PRC 47, 2242 (1993)



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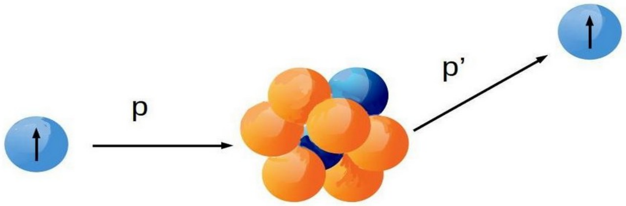
Proton Nucleus Elastic Scattering from 0^+ nuclei in Leading Order



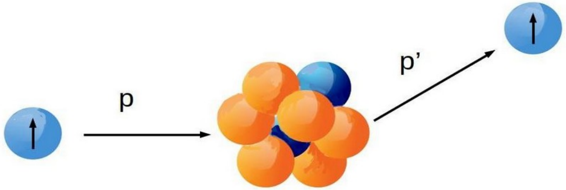
Nucleus described e.g. by NSCM calculation.
One-Body Density Matrix (OBDM)

$$\rho(\boldsymbol{p}, \boldsymbol{p}') = \left\langle \phi' \left| \sum_{i=1}^{A} \delta^3(\boldsymbol{p}_i - \boldsymbol{p}) \delta^3(\boldsymbol{p}_i' - \boldsymbol{p}') \prod_{j \neq i}^{A} \delta^3(\boldsymbol{p}_j - \boldsymbol{p}_j') \right| \phi \right\rangle$$

Nonlocal in p and p' → need to remove center-of-mass motion

Burrows et al. PRC97, 024325 (2018)

Proton Nucleus Elastic Scattering from 0^+ nuclei in Leading Order



What is wrong with this picture?

Nucleus described e.g. by NSCM calculation.

One-Body Density Matrix (OBDM)

$$\rho(\boldsymbol{p}, \boldsymbol{p}') = \left\langle \phi' \left| \sum_{i=1}^{A} \delta^3(\boldsymbol{p}_i - \boldsymbol{p}) \delta^3(\boldsymbol{p}_i' - \boldsymbol{p}') \prod_{j \neq i}^{A} \delta^3(\boldsymbol{p}_j - \boldsymbol{p}_j') \right| \phi \right\rangle$$

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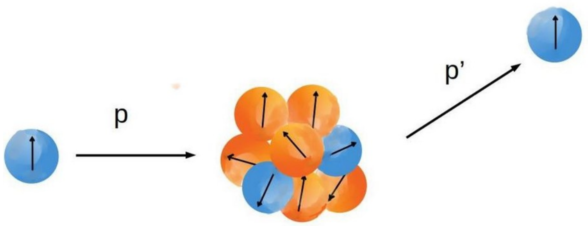
Proton Nucleus Elastic Scattering from 0+ nuclei in Leading Order



In the NN interactions both nucleons carry spin = ½

→the OBDM must contain the spin of the single nucleon

$$\rho^{S}_{M_s}(\boldsymbol{p},\boldsymbol{p}') = \left\langle \phi'(p_i', p_2, p_3, ..., p_A) \middle| \sum_i^A \delta^3(\boldsymbol{p}-\boldsymbol{p}_i)\delta^3(\boldsymbol{p}'-\boldsymbol{p}_i')\sigma^{S}_{M_s} \middle| \phi(p_i, p_2, p_3, ..., p_A) \right\rangle$$

where

$$\begin{aligned} S=0 \quad &: \quad \sigma_0^0 = 1 \\ S=1 \quad &: \quad \sigma_0^1 = \hat{\sigma}_z \\ &: \quad \sigma_{-1}^1 = \frac{1}{\sqrt{2}}(\hat{\sigma}_x - i\hat{\sigma}_y) \\ &: \quad \sigma_1^1 = -\frac{1}{\sqrt{2}}(\hat{\sigma}_x + i\hat{\sigma}_y) \end{aligned}$$

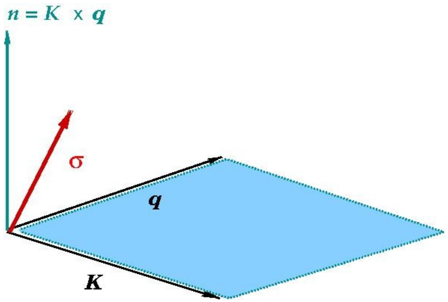
← Scalar density

NN amplitude in Wolfenstein represenation:

L. Wolfenstein and J. Ashkin, Phys. Rev. 85, 947 (1952)

with $q = k' - k$

$K_{NN} = \frac{1}{2}(k' + k)$



$$
\begin{aligned}
\overline{M}(\boldsymbol{q},\mathcal{K}_{NN},\epsilon) = \; & A(\boldsymbol{q},\mathcal{K}_{NN},\epsilon)\mathbf{1}\otimes\mathbf{1} \\
& + iC(\boldsymbol{q},\mathcal{K}_{NN},\epsilon)\,(\boldsymbol{\sigma}^{(0)}\cdot\hat{\boldsymbol{n}})\otimes\mathbf{1} \\
& + iC(\boldsymbol{q},\mathcal{K}_{NN},\epsilon)\,\mathbf{1}\otimes(\boldsymbol{\sigma}^{(i)}\cdot\hat{\boldsymbol{n}}) \\
& + M(\boldsymbol{q},\mathcal{K}_{NN},\epsilon)(\boldsymbol{\sigma}^{(0)}\cdot\hat{\boldsymbol{n}})\otimes(\boldsymbol{\sigma}^{(i)}\cdot\hat{\boldsymbol{n}}) \\
& + [G(\boldsymbol{q},\mathcal{K}_{NN},\epsilon) - H(\boldsymbol{q},\mathcal{K}_{NN},\epsilon)]\,(\boldsymbol{\sigma}^{(0)}\cdot\hat{\boldsymbol{q}})\otimes(\boldsymbol{\sigma}^{(i)}\cdot\hat{\boldsymbol{q}}) \\
& + [G(\boldsymbol{q},\mathcal{K}_{NN},\epsilon) + H(\boldsymbol{q},\mathcal{K}_{NN},\epsilon)]\,(\boldsymbol{\sigma}^{(0)}\cdot\hat{\mathcal{K}})\otimes(\boldsymbol{\sigma}^{(i)}\cdot\hat{\mathcal{K}}) \\
& + D(\boldsymbol{q},\mathcal{K}_{NN},\epsilon)\left[(\boldsymbol{\sigma}^{(0)}\cdot\hat{\boldsymbol{q}})\otimes(\boldsymbol{\sigma}^{(i)}\cdot\hat{\mathcal{K}}) + (\boldsymbol{\sigma}^{(0)}\cdot\hat{\mathcal{K}})\otimes(\boldsymbol{\sigma}^{(i)}\cdot\hat{\boldsymbol{q}})\right]
\end{aligned}
$$

Blue = projectile
Red = target nucleon

Evaluate scalar products with $\sigma^{(i)}$

$$\sigma^{(i)}\cdot\hat{n}$$

$$\boxed{\begin{array}{c}\sigma^{(i)}\cdot\widehat{K} \\ \sigma^{(i)}\cdot\hat{q}\end{array}}$$

Vanishes when evaluating
In 0^+ ground state

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Spin-projected momentum distribution $\left(\boldsymbol{\sigma}^{(i)} \cdot \hat{\boldsymbol{n}}\right)$ (in 0^+ ground state)

$$S_n(\boldsymbol{p}', \boldsymbol{p}) = \sum_{M_s} (-1)^{-M_s} \hat{\boldsymbol{n}}^1_{-M_s} \left\langle \phi' \left| \sum_{i=1}^{A} \delta^3(\boldsymbol{p}_i - \boldsymbol{p}) \delta^3(\boldsymbol{p}'_i - \boldsymbol{p}') \sigma^1_{M_s} \right| \phi \right\rangle$$

Evaluation based on NCSM matrix elements

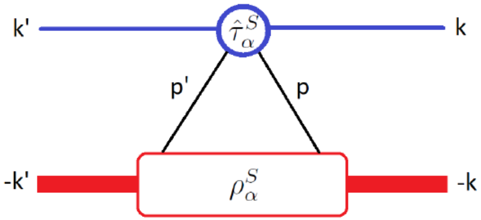
Change of variables to remove CoM

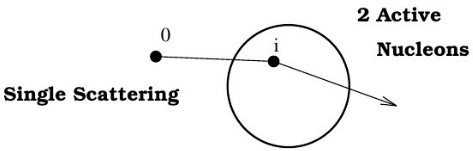
$$\begin{aligned} S_n(\boldsymbol{q}, \boldsymbol{\mathcal{K}}) \quad = \quad & -i\sqrt{3} \sum_{nsljn'l'j'} \langle n_{\mathcal{K}} l_{\mathcal{K}}, n_q l_q : K_l | n'l', nl : K_l \rangle_{d=1} \\ & \sum_{M_s=-1,1} \mathcal{Y}^{*l_q l_{\mathcal{K}}}_{1-M_s}(\hat{\boldsymbol{q}}, \hat{\boldsymbol{\mathcal{K}}}) \\ & (-1)^{-l} \hat{j}' \hat{j} \begin{Bmatrix} l' & l & K_l \\ s & s & 1 \\ j' & j & K \end{Bmatrix} R_{n_q l_q}(q) R_{n_{\mathcal{K}} l_{\mathcal{K}}}(\mathcal{K}) \\ & \left\langle AJ'\lambda' \left|\left| (a^\dagger_{n'l'sj'} \tilde{a}_{nlsj})^{(K)} \right|\right| AJ\lambda \right\rangle \end{aligned}$$

$$\begin{aligned} \vec{q} &= \vec{p'} - \vec{p} \\ \vec{\mathcal{K}} &= \frac{1}{2}(\vec{p'} + \vec{p}) \end{aligned}$$

Derivation in Burrows et al.
PRC 102, 034606 (2020)

Computing the leading order effective potential $U^{(1)} \approx \Sigma^{A}_{i=0}\ \tau_{0i}$





$$
\begin{aligned}
&\widehat{U}_p(\boldsymbol{q}, \boldsymbol{\mathcal{K}_{NA}}, \epsilon) \\
&= \sum_{\alpha=p,n} \int d^3\mathcal{K}\eta\,(\boldsymbol{q}, \boldsymbol{\mathcal{K}_{NA}}, \epsilon)\, \boldsymbol{A_{p,\alpha}} \left(\boldsymbol{q}, \frac{1}{2}\left(\frac{A+1}{A}\boldsymbol{\mathcal{K}_{NA}} + \boldsymbol{\mathcal{K}}\right), \epsilon\right) \boxed{\rho^{S=0}_\alpha(\boldsymbol{P'}, \boldsymbol{P})} \quad \text{scalar density} \\
&+ i(\boldsymbol{\sigma}^{(0)} \cdot \hat{\boldsymbol{n}}) \sum_{\alpha=N,Z} \int d^3\mathcal{K}\eta\,(\boldsymbol{q}, \boldsymbol{\mathcal{K}_{NA}}, \epsilon)\, \boldsymbol{C_{p,\alpha}} \left(\boldsymbol{q}, \frac{1}{2}\left(\frac{A+1}{A}\boldsymbol{\mathcal{K}_{NA}} + \boldsymbol{\mathcal{K}}\right), \epsilon\right) \boxed{\rho^{S=0}_\alpha(\boldsymbol{P'}, \boldsymbol{P})} \\
&+ i \sum_{\alpha=N,Z} \int d^3\mathcal{K}\eta\,(\boldsymbol{q}, \boldsymbol{\mathcal{K}_{NA}}, \epsilon)\, \boldsymbol{C_{p,\alpha}} \left(\boldsymbol{q}, \frac{1}{2}\left(\frac{A+1}{A}\boldsymbol{\mathcal{K}_{NA}} + \boldsymbol{\mathcal{K}}\right), \epsilon\right) \boxed{S_{n,\alpha}(\boldsymbol{P'}, \boldsymbol{P})} \cos\beta \\
&+ i(\boldsymbol{\sigma}^{(0)} \cdot \hat{\boldsymbol{n}}) \sum_{\alpha=N,Z} \int d^3\mathcal{K}\eta\,(\boldsymbol{q}, \boldsymbol{\mathcal{K}_{NA}}, \epsilon)\, \boldsymbol{M_{p,\alpha}} \left(\boldsymbol{q}, \frac{1}{2}\left(\frac{A+1}{A}\boldsymbol{\mathcal{K}_{NA}} + \boldsymbol{\mathcal{K}}\right), \epsilon\right) \boxed{S_{n,\alpha}(\boldsymbol{P'}, \boldsymbol{P})} \cos\beta
\end{aligned}
$$

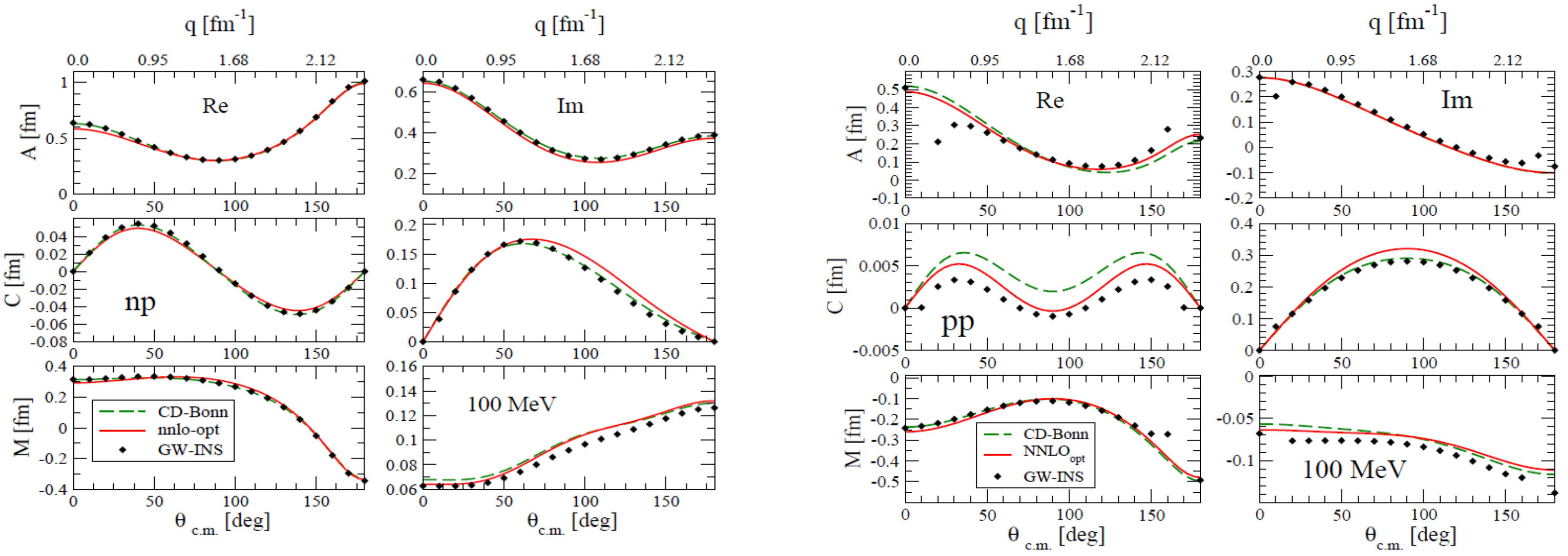
with $\quad \mathcal{P}' = \left(\mathcal{K} - \frac{A-1}{A}\frac{q}{2}\right)$ and $\mathcal{P} = \left(\mathcal{K} + \frac{A-1}{A}\frac{q}{2}\right)$

Details of implementation designed for energies ≥ 100 MeV

Wolfenstein Amplitudes A, C, M

NNLO_{opt} fitted to E_{lab}=125 MeV

→ max. momentum transfer ≈ 2.45 fm^{-1}

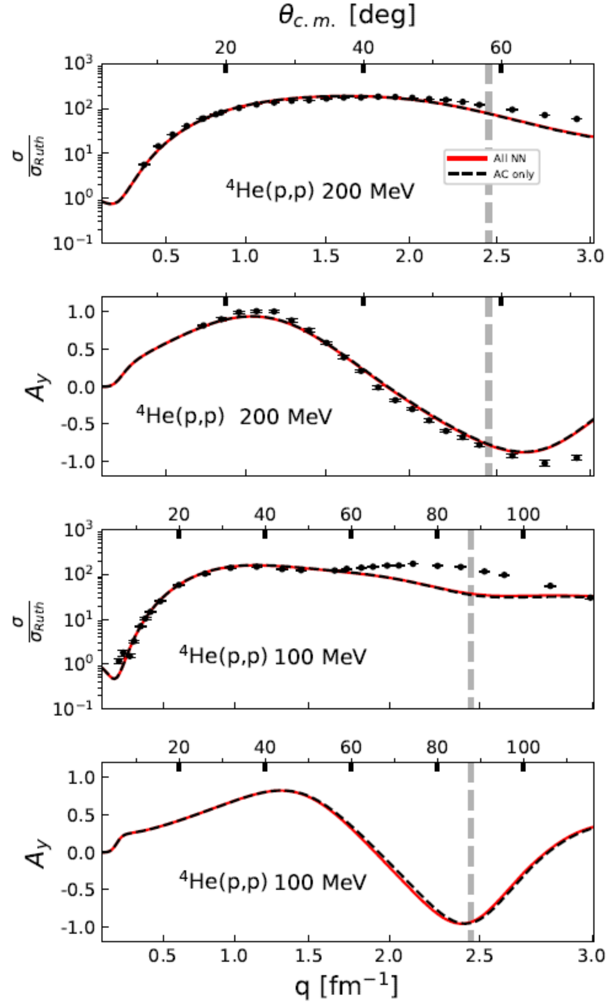


NNLO_{opt} A. Ekström, G. Baardsen, C. Forssén, G. Hagen, M. Hjorth-Jensen, G. R. Jansen, R. Machleidt, W. Nazarewicz, *et al.*, Phys. Rev. Lett. **110**, 192502 (2013).

CD-Bonn R. Machleidt, Phys. Rev. **C63**, 024001 (2001)

GW-INS R. L. Workman, W. J. Briscoe, and I. I. Strakovsky, Phys. Rev. **C94**, 065203 (2016).

^{4}He N_{max}=18



$$\vec{q}_{nn} = \vec{q}_{nA} = \vec{q}$$

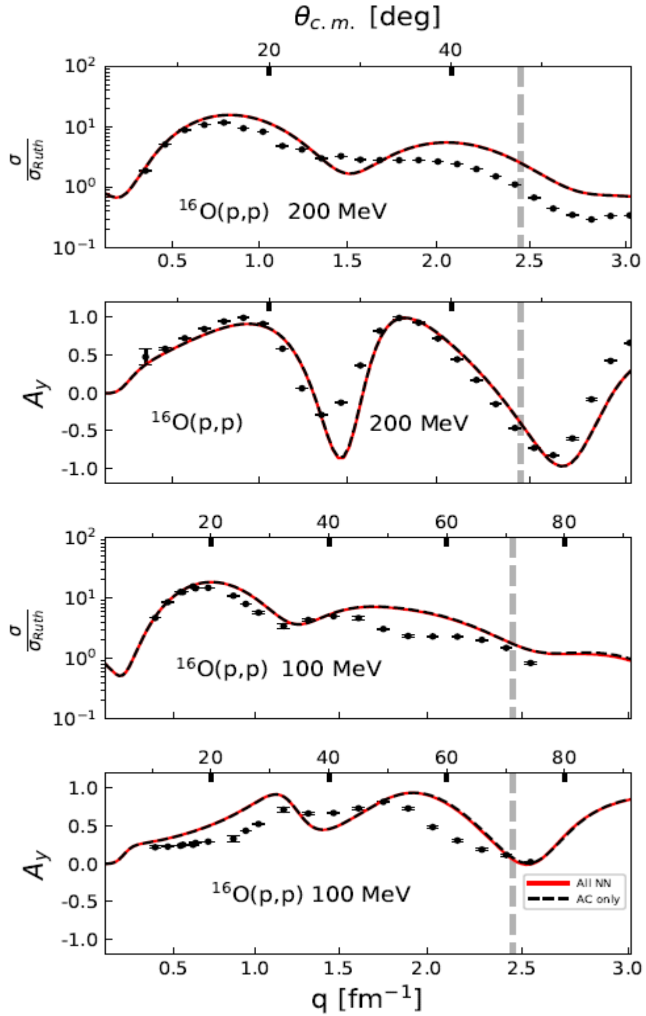
$$q \approx 480\ \text{MeV} = 2.45\ \text{fm}^{-1}$$

closed-shell nuclei

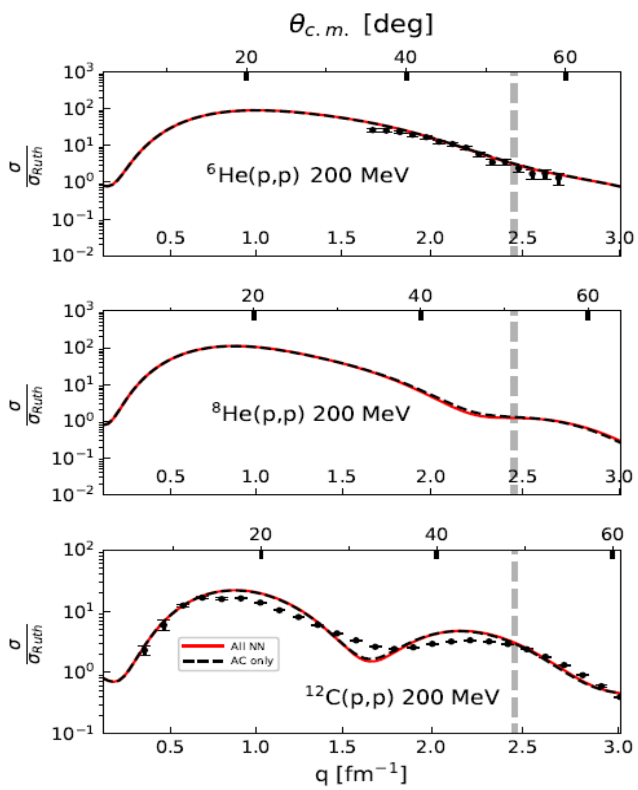
NNLO$_{opt}$
Chiral interaction

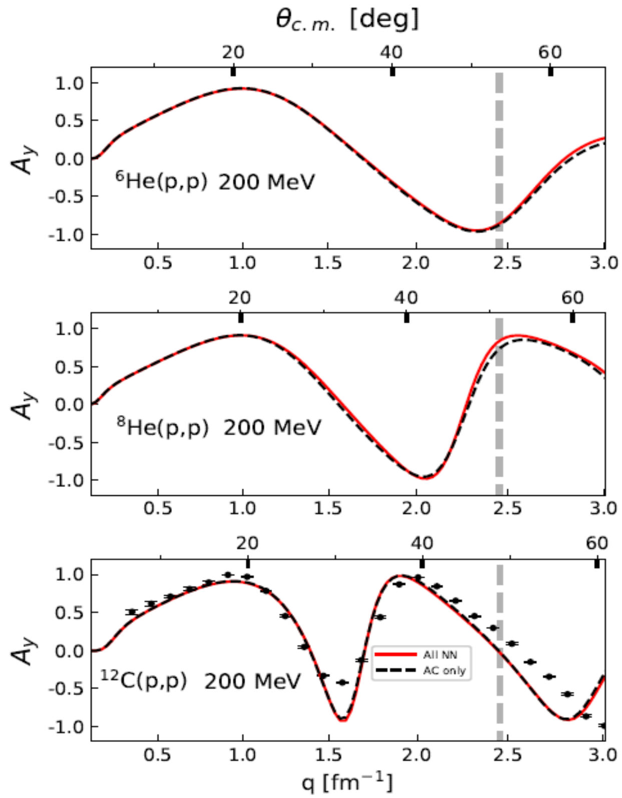
$\hbar\omega$=20

N_{max}=10 ^{16}O

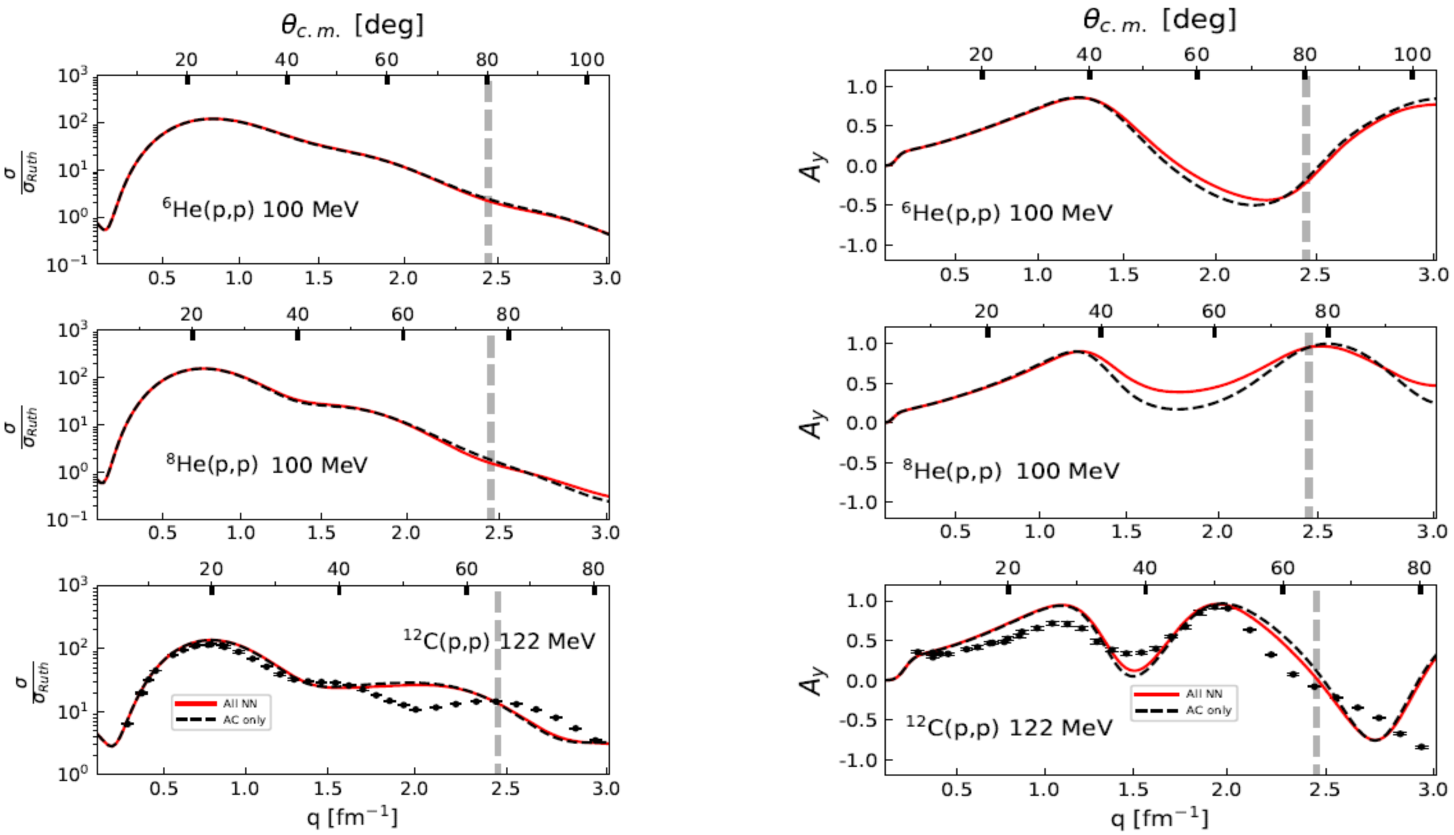


Open-shell nuclei at 200 MeV

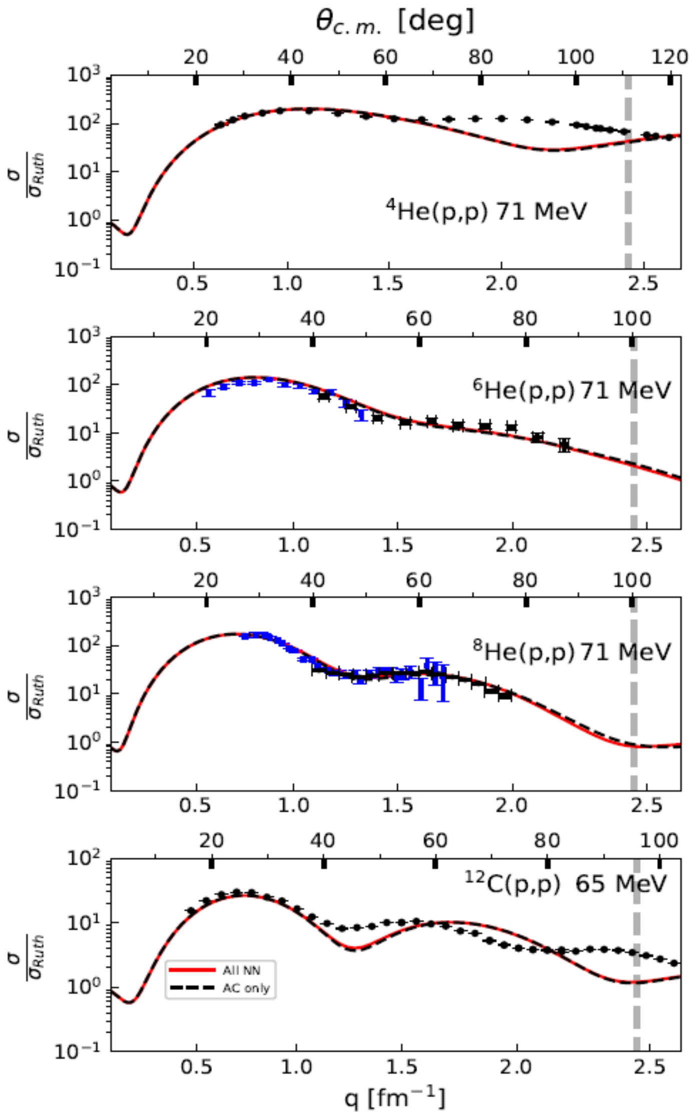




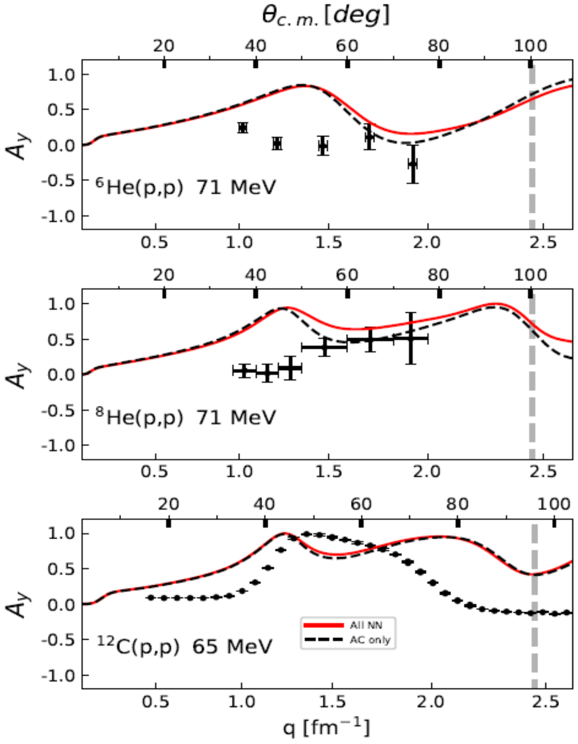
Open-shell nuclei at 100 MeV

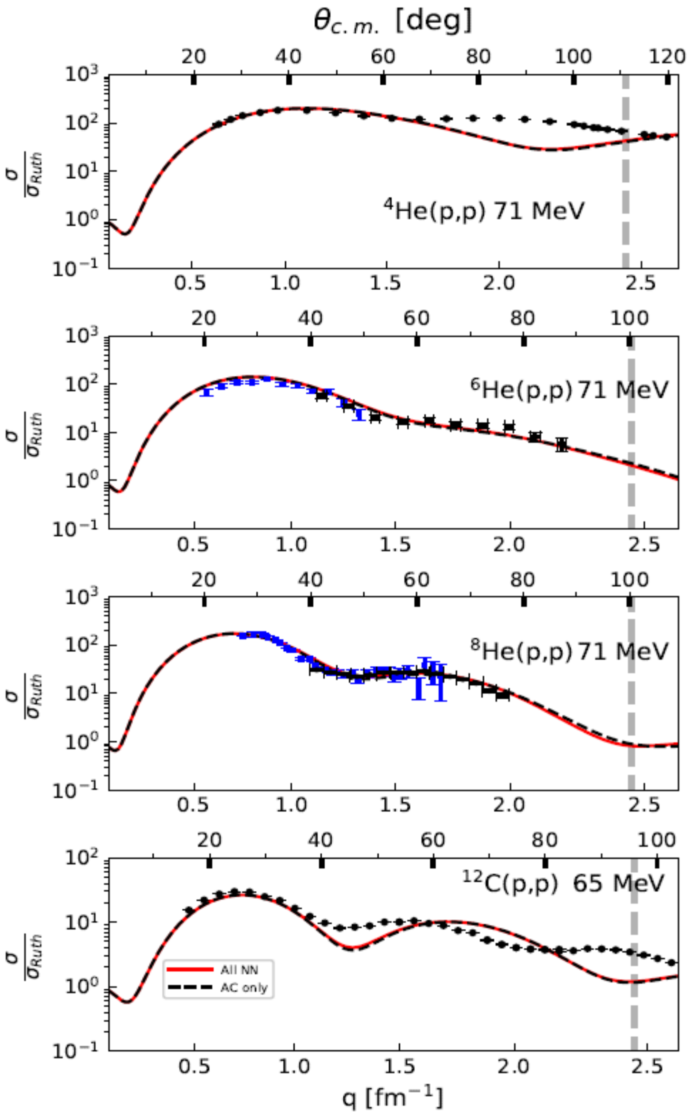


Energies lower than 100 MeV



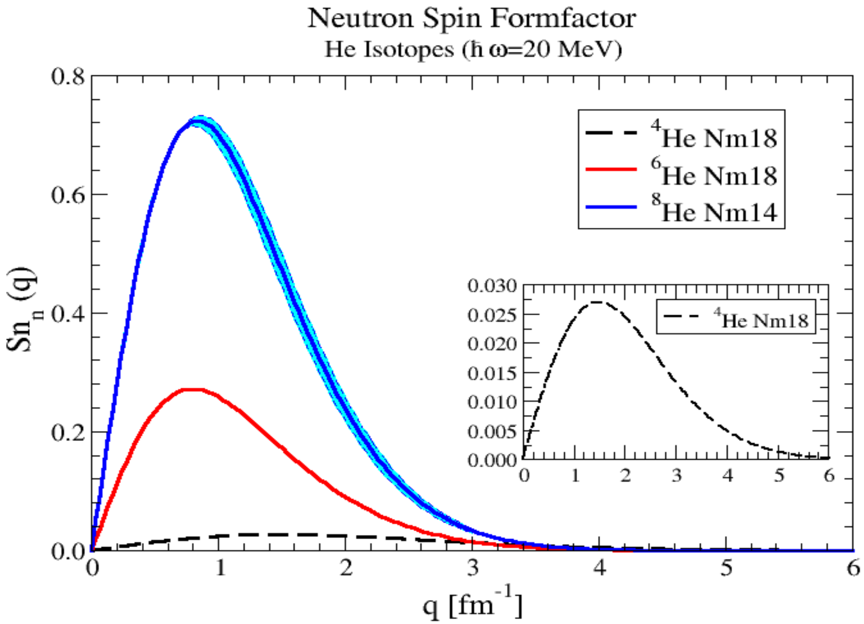
Energies lower than 100 MeV





Can we learn more from the ground state spin projected momentum distribution?

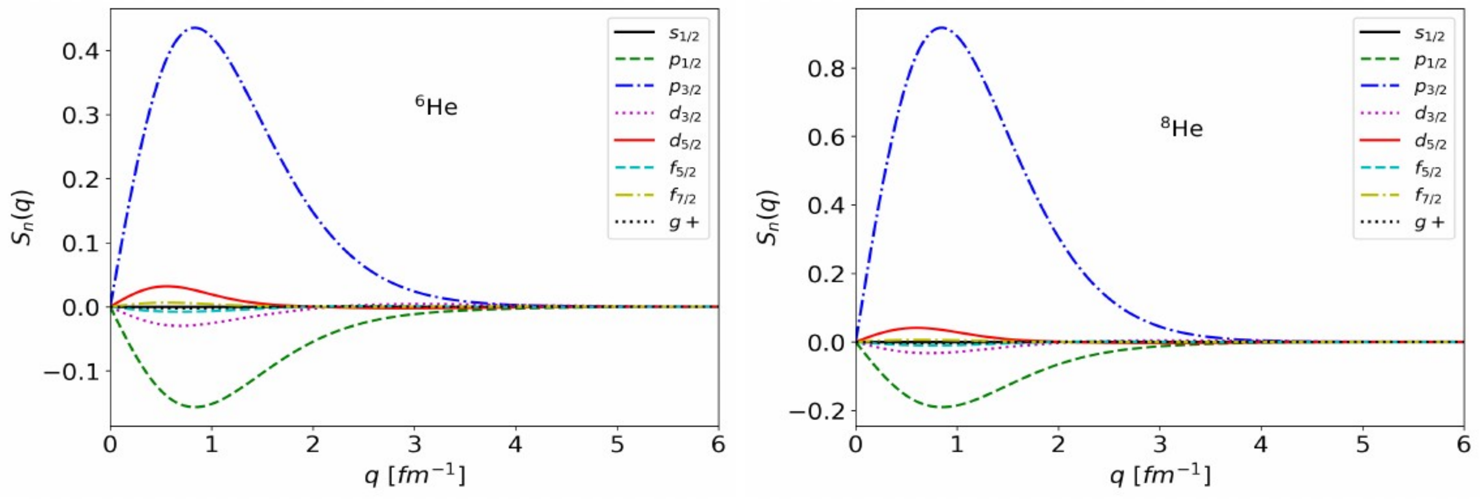
Define: $$S_n(\mathbf{q}) = \int d^3K S_n(\mathbf{q}, \mathbf{K})$$ == “spin form factor”



With NNLO$_{opt}$
Chiral interaction

Can we learn more from the ground state spin projected momentum distribution?

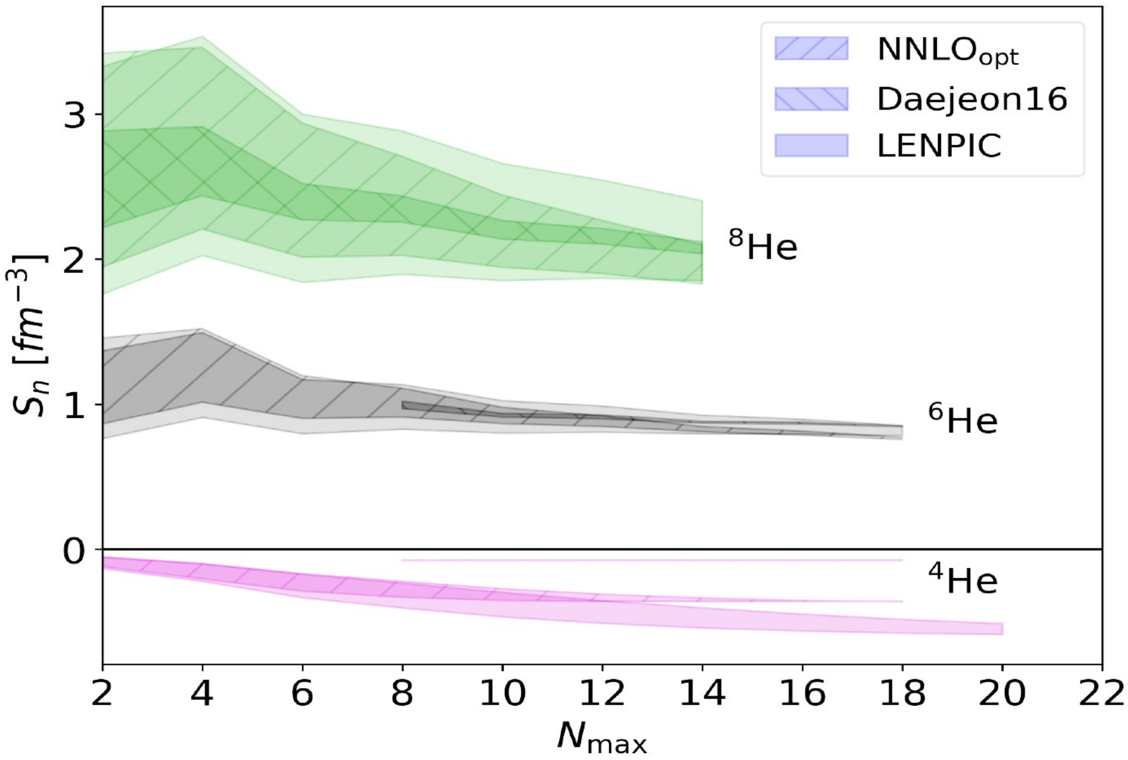
Neutron distributions according to sub-shell contributions



The sign of $S_n(q)$ depends on alignment of orbital angular momentum and spin

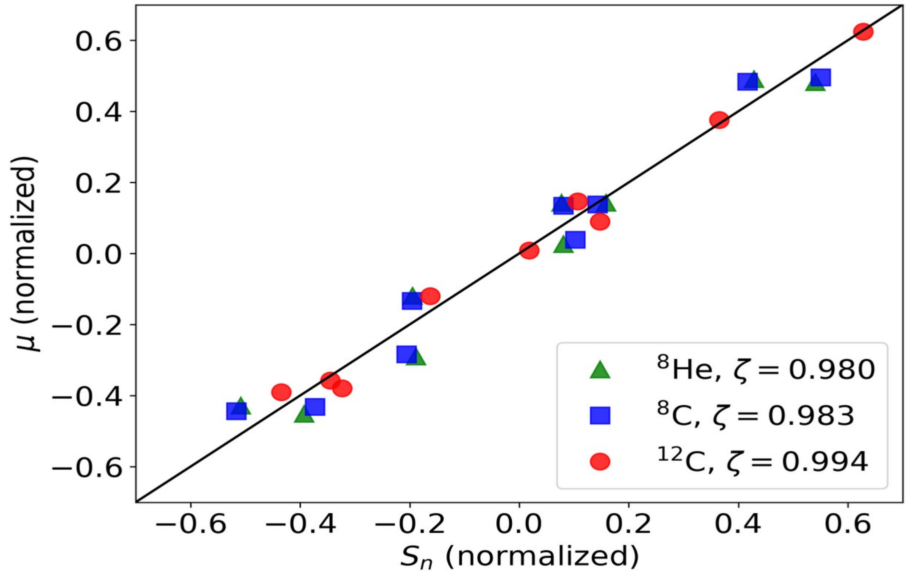
Integrated spin-projected neutron momentum distributions for Helium isotopes

$$S_n := \int dq\, q^2 S_n(q)$$



Correlation with observable:

- Total spin-projected momentum distribution in 0+ ground state
- Magnetic moment in 2+ excited state



R.B. Baker et al. in preparation (stay tuned)

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p+A and n+A effective interactions ("optical potentials")

- Renewed urgency in reaction theory community for microscopic input to e.g. (d,p) reaction models .
- Most likely complementary approaches needed for different energy regimes

Consistent approach to p+A effective interaction is possible.

- **In the multiple scattering approach the leading order term can be calculated consistently *ab initio***
  (spin of projectile and struck target nucleon treated consistently)
- **Effect of spin of the struck nucleon visible in spin-observables for N≠Z nuclei in He isotopes**
- Effect in other isotope chains?
  Connection of spin form factors to observables?

- Dependence on NN forces employed
- Refinement of calculation of leading order term for energies below 100 MeV
- Work paves the road to consider inelastic reactions