Effective field theory approach for low energy elastic scattering of protons

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Contents

1. Introduction:
Motivation

- Resonant capture reaction $\mathrm{A(X,\gamma)B}$

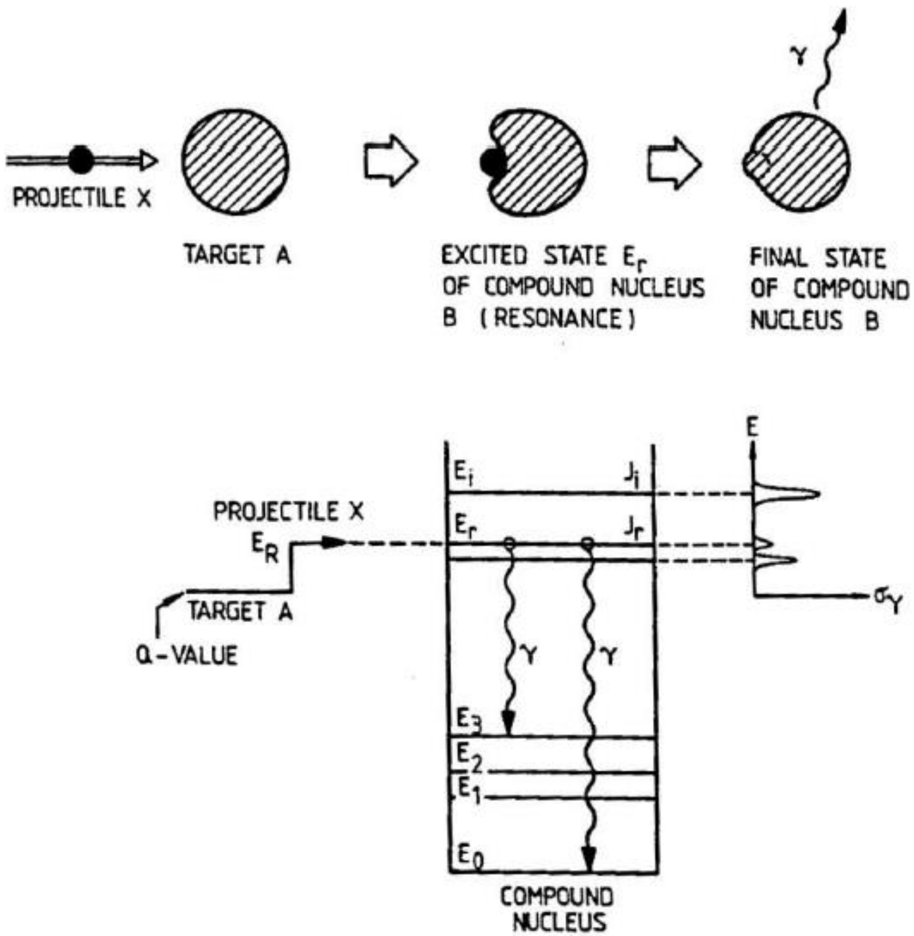


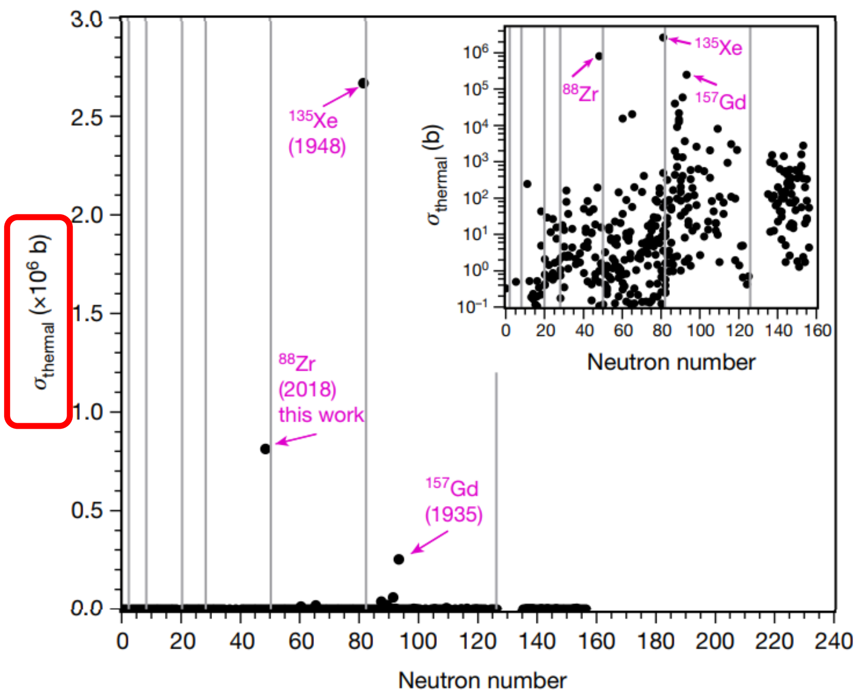
Figure from D.D. Clayton “Principles of stellar evolution and nucleosynthesis”.

1. Introduction:
Motivation

- J.A. Shusterman et al., Nature **565,** 328–330 (2019)

The surprisingly large neutron capture cross-section of ^{88}Zr

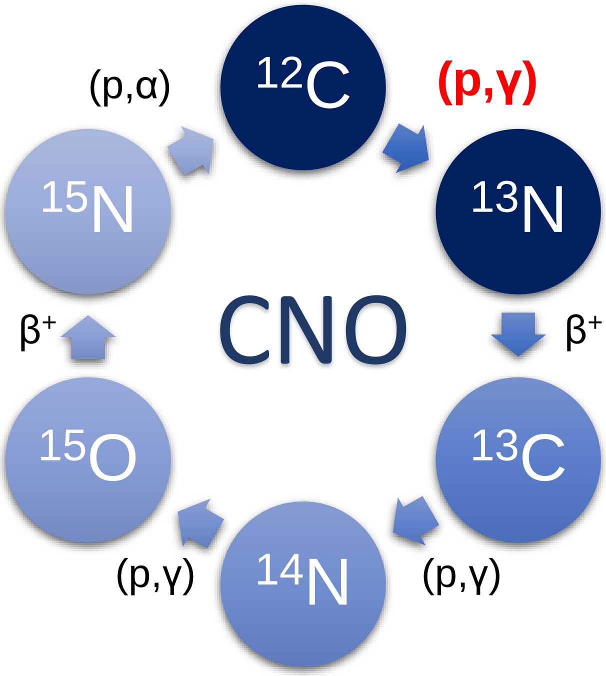
Jennifer A. Shusterman[1,2,3]*, Nicholas D. Scielzo[1], Keenan J. Thomas[1], Eric B. Norman[4], Suzanne E. Lapi[5], C. Shaun Loveless[5], Nickie J. Peters[6], J. David Robertson[6], Dawn A. Shaughnessy[1] & Anton P. Tonchev[1]



1. Introduction:

Motivation I: elastic p+^{12}C scattering

- Radiative proton capture reaction, $^{12}\mathrm{C}(\mathrm{p},\gamma)^{13}\mathrm{N}$



✓ first reaction in the CNO cycle

1. Introduction:
Motivation II: *NN* scattering

- Regularization
  - To handle the divergences in
    - loop integrals
    - Lippmann-Schwinger Equation ex) $T = V + \int_0^\infty d\vec{l}\, V G_0 T, \quad \langle \vec{p}|V|\vec{p}'\rangle = C_0$

- Momentum cutoff regularization[1,2] $\int_0^\infty dl \;\rightarrow \int_0^{\Lambda} dl = \int_0^\infty dl\, R_\Lambda(l)$
  - Nicely meets the EFT philosophy = separation of scales

✓ **But it often breaks the unitarity of systems** !

- Relation between the EFT and R-matrix
  - Λ: momentum cutoff (EFT)
  - a: channel radius (R-matrix)

✓ In this work, we study of relation between the EFT and the R-matrix.

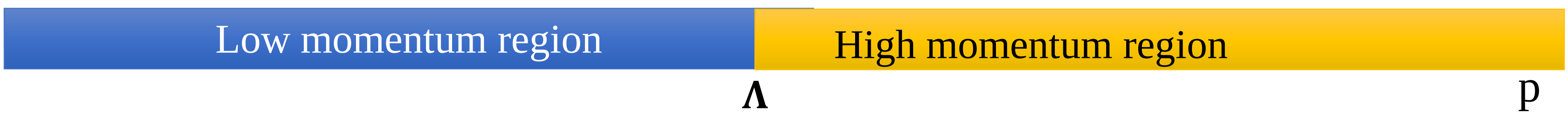
1. Introduction:
Effective Field Theory

- EFT provides a general approach to calculate low-energy observables by scale separation[1),2)]

- Key elements of an EFT

• Scale separation

– observables at typical momentum scale Q
– short-range physics at scale Λ, where $\Lambda \gg \mathrm{Q}$
– Q/ Λ ~ expansion parameter



• Systematic expansion in Q/Λ:

– effective Lagrangian:

$$\mathcal{L} = \sum_{\nu,i} c_{\nu,i}\hat{O}_{\nu,i} \quad \text{where } \hat{O}_{\nu,i} \sim \text{order of the } (Q/\Lambda)^{\nu}$$

– a limited number of low-energy constants (LECs) enter at a given EFT order
– predict observables at Q-scale with controlled uncertainties at each order

[1] S. Weinberg, Nucl. Phys. B 363 (1991). 3. [2] S. Weinberg, Nucl. Phys. Phys. Lett. B 295 (1992). 114.

1. Introduction:
Effective Field Theory

- Low-energy constant (LECs)

<u>Low energy *NN* scattering</u>

$$L^{EFT} = N^{+} i\partial_t N - N^{+}\frac{\nabla^2}{2m_N}N - \frac{1}{2}C_0(N^{+}N)^2 - \frac{1}{2}C_2(N^{+}\nabla^2 N)(N^{+}N) + h.c. + \dots$$

Low energy constant (LECs)

✓ Contain the information of high momentum dynamics

✓ Fit to the experimental data

1. Introduction:
Effective Field Theory

- Low-energy Effective field theory (EFT)
  - Power Counting (Counting rule)[1)]

Here T is some transition amplitude.

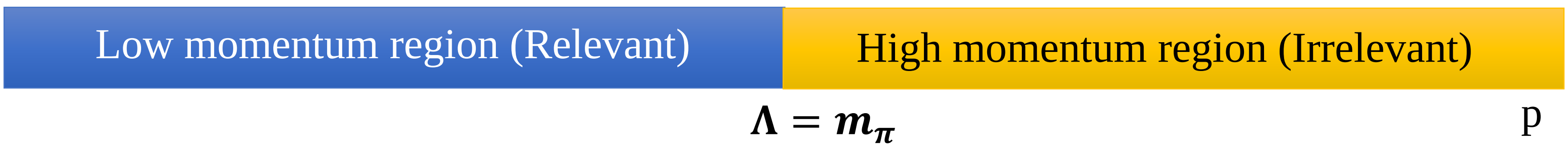
$$\mathrm{T}_{\mathrm{EFT}} = coef \cdot \left[1 + \frac{Q}{\Lambda} + \left(\frac{Q}{\Lambda}\right)^2 + \left(\frac{Q}{\Lambda}\right)^3 + \left(\frac{Q}{\Lambda}\right)^4 + \cdots \right]$$

Estimate uncertainty

[1] S. Weinberg, Physica A 96, 327 (1979); Phys. Lett. B 91, 51 (1980).

1. Introduction:
Pionless EFT

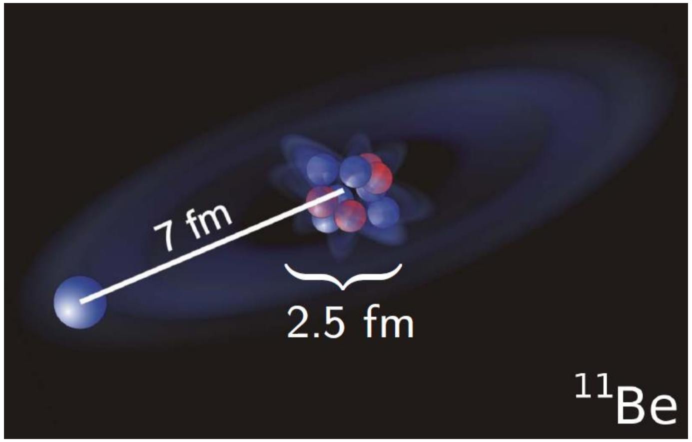
- Pionless Effective Field Theory (EFT)
  - Low-energy EFT – pion = Pionless EFT
    - $Q \sim$ small momentum scale in the system
    - $\Lambda \sim$ pion mass (m_π)



1. Introduction:
Halo/Cluster EFT

- Halo/Cluster Effective Field Theory(EFT) [1,2]
  - degree of freedom: core + valence nucleons
    - $Q \sim \sqrt{mS_n}$
    - $\Lambda \sim \sqrt{mE_C^*}$

S_n: neutron separation energy
E_C^*: core excitation energy



  - scale separation
    - $Q \ll \Lambda$ → systematic expansion in observables
    - Short-range effects are included in LECs.

[1] C.A. Bertulani, et al., Nucl. Phys. A 712 (2002) 37-58. [2] R.Higa, et al., Nucl. Phys. A 809 (2008) 171-188

1. Introduction:
Effective Range Expansion (ERE)

- LECs can be determined by effective range parameter.

Low energy NN scattering

$$k\ \cot\delta = -\frac{1}{\boldsymbol{a_0}} + \frac{1}{2}\boldsymbol{r_0}k^2 + \cdots$$

Effective Range Parameters

a_0: scattering length
r_0 : effective range

$$S = e^{2i\delta} = 1 + \frac{2ik}{\boldsymbol{k\cot\delta} - ik} = 1 + ik\frac{m_N}{2\pi}\boldsymbol{T}$$

from EFT

<ERE> $$iT^{({}^1S_0)} = \frac{4\pi}{m_N}\frac{i}{-\frac{1}{a_0} - ik}$$

<EFT> $$iT^{({}^1S_0)} = \frac{4\pi}{m_N}\frac{i}{-\frac{4\pi}{m_N C_0} - ik}$$

$$C_0 = \frac{4\pi}{m_N}a_0$$

✓ LEC can be determined by effective range parameter

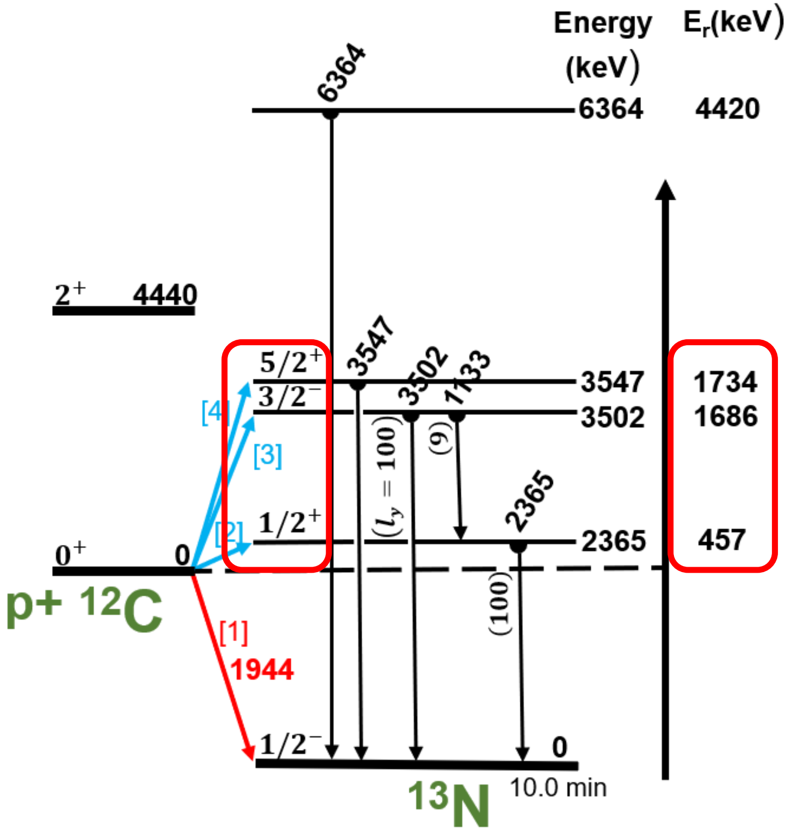
[1] H.A. Bethe, Phys. Rev. **76**, 38 (1949). [2] R.Machleidt, Phys. Rev. C **63**, 024001 (2001).

Contents

2. Cluster EFT for elastic p+^{12}C scattering

Power Counting

- Separation of scales[1,2]:
  - small momentum scale of the system
    $$Q \sim \sqrt{2\mu E_r} \sim 30, 60\ \mathrm{MeV}$$
  - large momentum scale of the system
    $$\Lambda \sim \sqrt{2\mu E_C^*} \sim 90\ \mathrm{MeV},\ \text{where } \Lambda \gg Q$$
    E_C^* = 4.439 MeV
  - expansion parameter in $Q/\Lambda \approx \frac{1}{3}$ or $\frac{1}{2}$



[1] C.A. Bertulani, et al., Nucl. Phys. A 712 (2002) 37-58. [2] R.Higa, et al., Nucl. Phys. A 809 (2008) 171-188

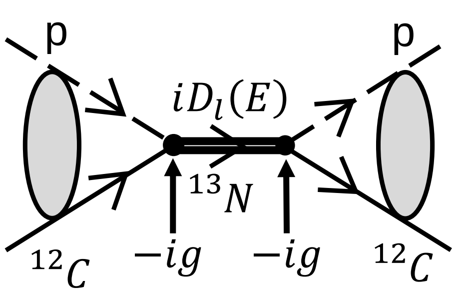
2. Cluster EFT for elastic p+^{12}C scattering
Lagrangian, Scattering amplitudes

- $$\mathcal{L} = p^\dagger\left(iD_t + \frac{\boldsymbol{D}^2}{2m_p}\right)p + c^\dagger\left(iD_t + \frac{\boldsymbol{D}^2}{2m_c}\right)c + \sum_{x=\frac{1}{2}^+,\frac{3}{2}^-,\frac{5}{2}^+} d_x^\dagger\left[\Delta_x + \sum_{n=0}^{N}\nu_{n,x}\left(iD_t + \frac{\boldsymbol{D}^2}{2m_{tot}}\right)^n\right]d_x$$
$$-\sum_{x=\frac{1}{2}^+,\frac{3}{2}^-,\frac{5}{2}^+} g_x\left[d_x^\dagger\left[p \underset{\nabla}{\leftrightarrow} c\right]_x + h.c.\right]$$ [1,2]

where p is the proton field with mass m_p, c is the ^{12}C field with mass m_c,
d_x are the dicluster field with mass m_{tot},
D_μ is a covariant derivative, N is 1 for s-, p- waves and 2 for d-waves,
Δ_x are mass difference, $\nu_{n,x} = \pm 1$
g_x are coupling constants when the dicluster field breaks up into a proton and a core,

$$\left[p \underset{\nabla}{\leftrightarrow} c\right]_{\frac{3}{2}^-} = \sum_{m_s,m_l} C^{\frac{3}{2}m}_{1m_l,\frac{1}{2}m_s}\left(p \underset{\nabla}{\leftrightarrow} c\right) \text{ with } p \underset{\nabla}{\leftrightarrow} c = p\left(\frac{m_c\overleftarrow{\nabla} - m_p\overrightarrow{\nabla}}{m_{tot}}\right)c$$

- elastic scattering amplitude for $l = 0, 1, 2$ channels



$$iT(E;\boldsymbol{k},\boldsymbol{k}') = iT_C(E) + iT_{SC}(E)$$

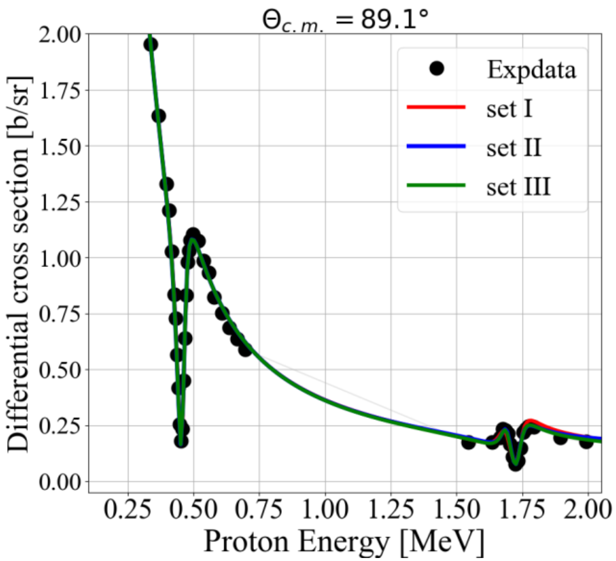
well known

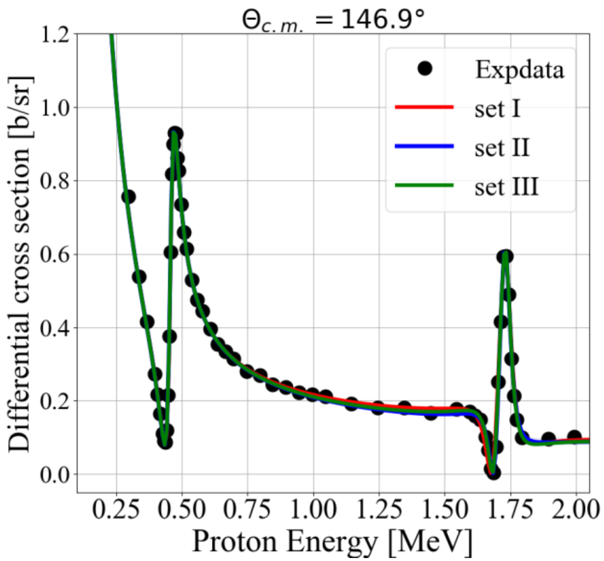
to be calculated

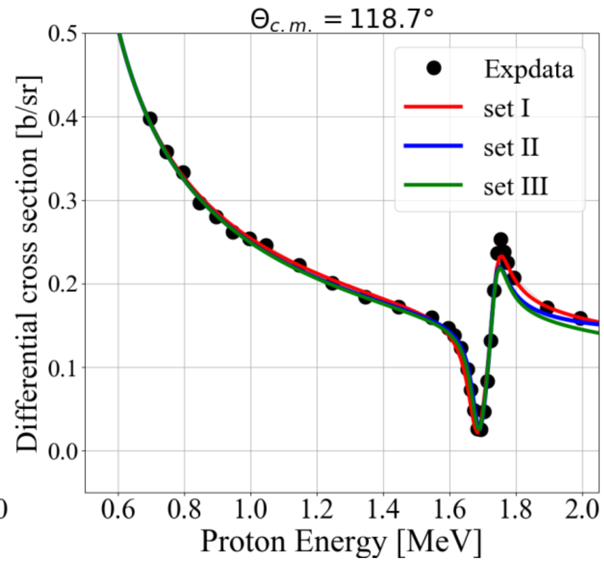
$T_C(E)$: Coulomb scattering amplitude
$T_{SC}(E)$: Coulomb-modified strong scattering amplitude

[1] R.Higa, et al., Nucl. Phys. A 809, 171 (2008) [2] P.F. Bedaque, et al., Phys. Lett. B 569, 159 (2003)

2. Cluster EFT for elastic p+^{12}C scattering
Results: differential cross sections





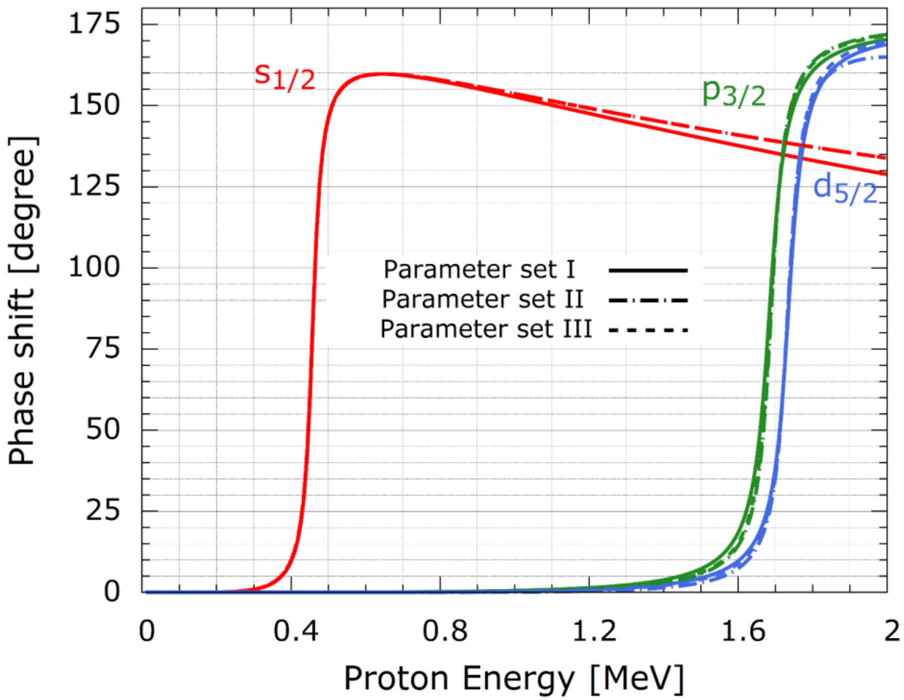


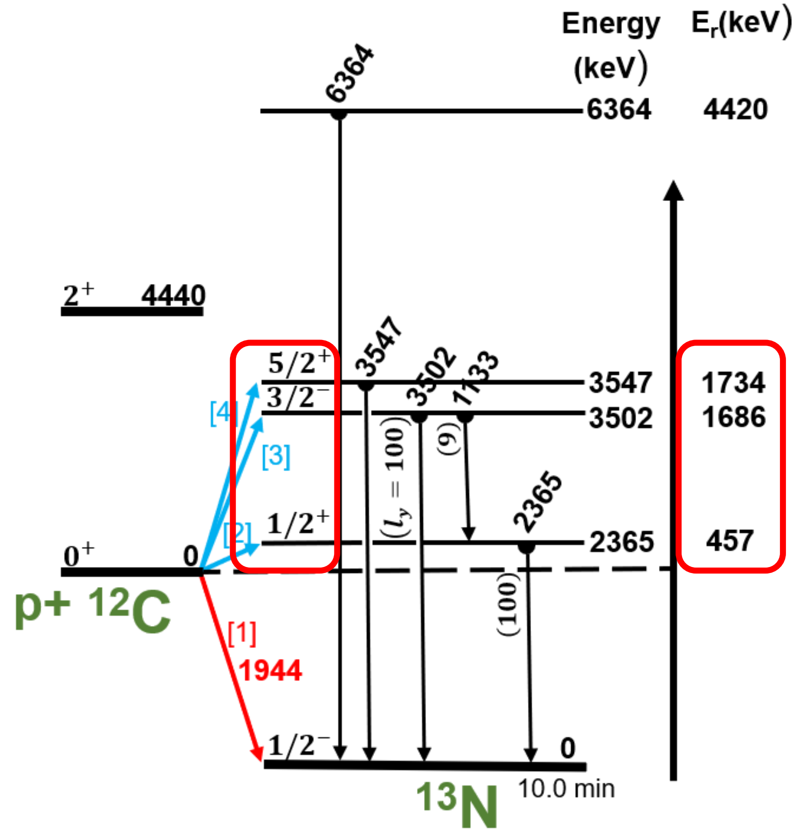
	Parameter Set I			Parameter Set II			Parameter Set III		
level	$S_{1/2}$	$P_{3/2}$	$D_{5/2}$	$S_{1/2}$	$P_{3/2}$	$D_{5/2}$	$S_{1/2}$	$P_{3/2}$	$D_{5/2}$
a (fm^{2L+1})	-285 ± 11	-17.2 ± 4.06	-47.8 ± 2.30	-323 ± 3.72	-13.0 ± 0.47	-10.9 ± 2.62	-323 ± 3.75	-13.7 ± 0.43	-41.4 ± 1.41
r (fm^{1-2L})	1.46 ± 0.02	-1.35 ± 0.77	-0.50 ± 0.04	1.53 ± 0.00	-2.01 ± 0.08	-4.39 ± 1.23	1.53 ± 0.00	-1.90 ± 0.07	-0.61 ± 0.02
P (fm^{3-2L})	-3.23 ± 0.89	3.16 ± 10.92	0.54 ± 0.97			-52.9 ± 17.1			

2. Cluster EFT for elastic p+^{12}C scattering

Results: phase shifts

- Phase shifts by the obtained parameters





2. Cluster EFT for elastic p+^{12}C scattering

Summary

- Differential cross section of the elastic $^{12}\mathrm{C}(\mathrm{p,p})^{12}\mathrm{C}$ scattering was calculated by using cluster effective field theory.

- The three resonance states ($\mathrm{J}_\pi = \frac{1}{2}^+, \frac{3}{2}^-, \frac{5}{2}^+$) of ^{13}N were considered.

- We obtained the effective range parameters for elastic $^{12}\mathrm{C}(\mathrm{p,p})^{12}\mathrm{C}$ scattering.

- Comparing the results calculated at leading order (LO) and next-to-leading order (NLO), we described the elastic $^{12}\mathrm{C}(\mathrm{p,p})^{12}\mathrm{C}$ scattering systematically.

- Future work
  - ^{12}C(p,γ)^{13}N reaction

    (1) Calculate radiative capture amplitude by using Cluster EFT

    (2) Obtain differential cross section → total cross section

    ✓ Reaction rates $N_A < \sigma v >$ at the characteristic stellar temperatures T

$$N_A < \sigma v > = \left(\frac{8}{\pi\mu}\right)^{\frac{1}{2}} \frac{N_A}{(kT)^{3/2}} \int_0^\infty \boldsymbol{\sigma(E)} E \exp\left(-\frac{E}{kT}\right) dE$$

    ✓ S-factor

$$S(E) = Eexp(2\pi\eta)\boldsymbol{\sigma_{tot}(E)}$$

Contents

3.1 Role of unitarity in EFT descriptions

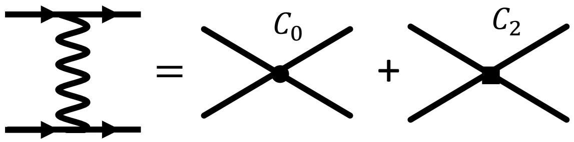
Motivation

- Regularization
  - To handle the divergence in
    - loop integrals
    - Lippmann-Schwinger Equation ex) $T = V + \int_0^\infty d\vec{l}\, V G_0 T, \quad \langle \vec{p} | V | \vec{p}' \rangle = C_0$
- Momentum cutoff regularization[1,2] $\int_0^\infty dl \rightarrow \int_0^\Lambda dl = \int_0^\infty dl\, R_\Lambda(l)$
  - Nicely meets the EFT philosophy = separation of scales
    - ✓ **But it often breaks the unitarity of systems** !
- Unitarity
  - Conservation of probability density
  - $S^\dagger S = 1$, S: S-matrix
  - If unitarity is violated, $S^\dagger S \neq 1$
- Our work
  - np $^1S_0$ scattering
  - Regulators
    - Unitarity-violating
    - Unitarity-preserving
  - Methods
    - EFT with only nucleons
    - Dibaryon field
  - Aim: To understand the phenomenological role of unitarity in EFT descriptions

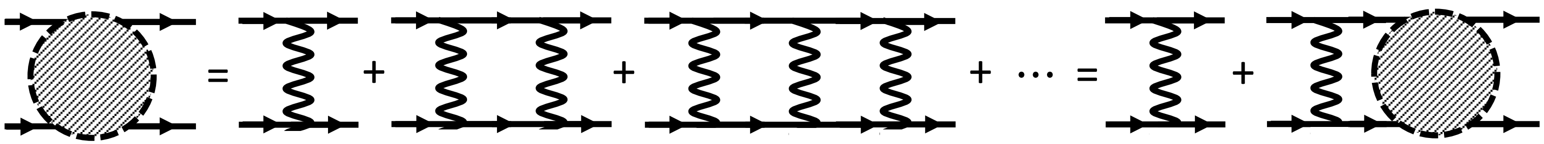
[1] S.R. Beane et al., Nucl. Phys. A **632**, 445 (1998). [2] D.R. Phillips et al., Ann. Phy. **263**, 255 (1998)

3.1 Role of unitarity in EFT descriptions
EFT with only nucleons

- $\mathcal{L} = N^\dagger\left(i\partial_t + \frac{\nabla^2}{2M}\right)N - \left[\frac{1}{2}C_0\left(N^\dagger N\right)^2 + \frac{1}{2}C_2\left(N^\dagger\nabla^2 N\right)\left(N^\dagger N\right) + \cdots\right] + h.c.$ [1)]

  - $V(p', p) \equiv$ 

$+ \cdots = C_0 + C_2(p^2 + p'^2)$



- Lippmann-Schwinger Equation
  - $T(p', p; E) = V(p', p) + \int \frac{d^3\vec{l}}{(2\pi)^3} V(p', l) \frac{M}{EM - l^2 + i\epsilon} T(l, p; E)\, \boldsymbol{R_\Lambda(l)}$

    $R_\Lambda(l)$: regulator with a momentum cutoff Λ

  - Momentum cutoff regularization of **loop calculation**

$$\int_0^\infty dl \rightarrow \int_0^\Lambda dl = \int_0^\infty dl\, R_\Lambda(l)$$

[1] S.R. Beane et al., Nucl. Phys. A **632**, 445 (1998).

3.1 Role of unitarity in EFT descriptions

EFT with only nucleons

- The np $^1S_0$ phase shifts:

$k \equiv \sqrt{ME}$: on-shell momentum

$$k\cot\delta = -\frac{4\pi}{M}\frac{1}{T(k,k;E)} + ik$$

$$= -\frac{4\pi}{M}\left(\frac{(1+\gamma_0 C_2)^2}{C_0 + 2C_2 k^2 - C_2^2(\gamma_2 - \gamma_0 k^2)}\right) + J_\Lambda(k) + \underline{ik(1 - R_\Lambda(k))}$$

$$J_\Lambda(k) \equiv \frac{2}{\pi} P\int_0^\infty dl\, R_\Lambda(l)\frac{l^2}{k^2 - l^2}, \quad \gamma_n \equiv M\int_0^\infty \frac{d^3\vec{l}}{(2\pi)^3} R_\Lambda(l) l^n \quad (n = 0,1)$$

- If $\boldsymbol{R_\Lambda(k)} \neq \mathbf{1}$, $k\cot\delta$: complex → δ: complex, $S^\dagger S \neq 1$ ($\because S = e^{2i\delta}$)

That is, unitarity is preserved if and only if $\boldsymbol{R_\Lambda(k) = 1}$!

3.1 Role of unitarity in EFT descriptions
Regulators

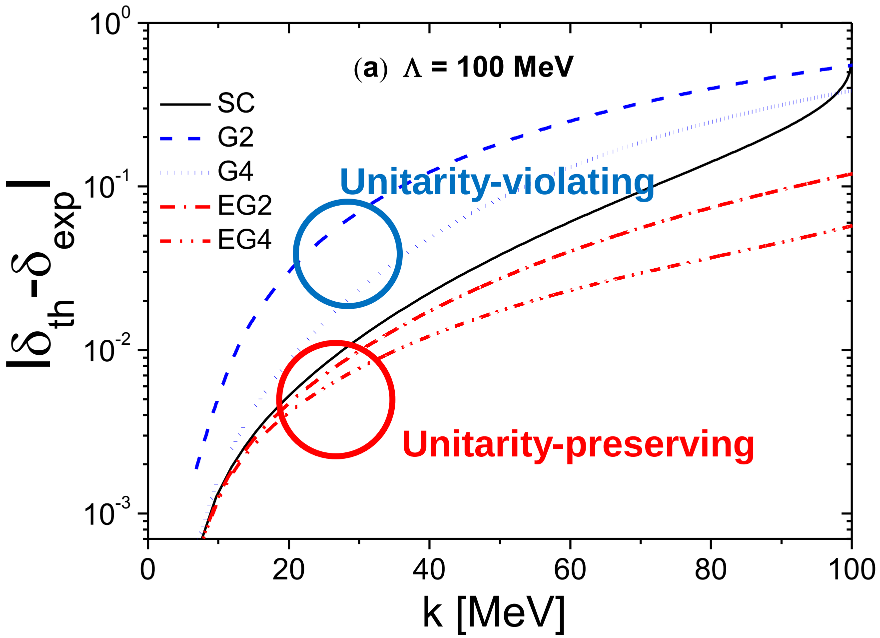
Unitarity	Condition	Regulator, $\mathrm{R}_\Lambda(l)$	Name
Unitarity-violating	$R_\Lambda(k) \neq 1$	$e^{-\frac{l^2}{\Lambda^2}}$	G2
		$e^{-\frac{l^4}{\Lambda^4}}$	G4
Unitarity-preserving	$R_\Lambda(k) = 1$	$e^{-\frac{(l^2-k^2)}{\Lambda^2}}$	EG2
		$e^{-\left(\frac{l^2-k^2}{\Lambda^2}\right)^2}$	EG4
		$\theta(\Lambda - l)$ in $k < \Lambda$	SC

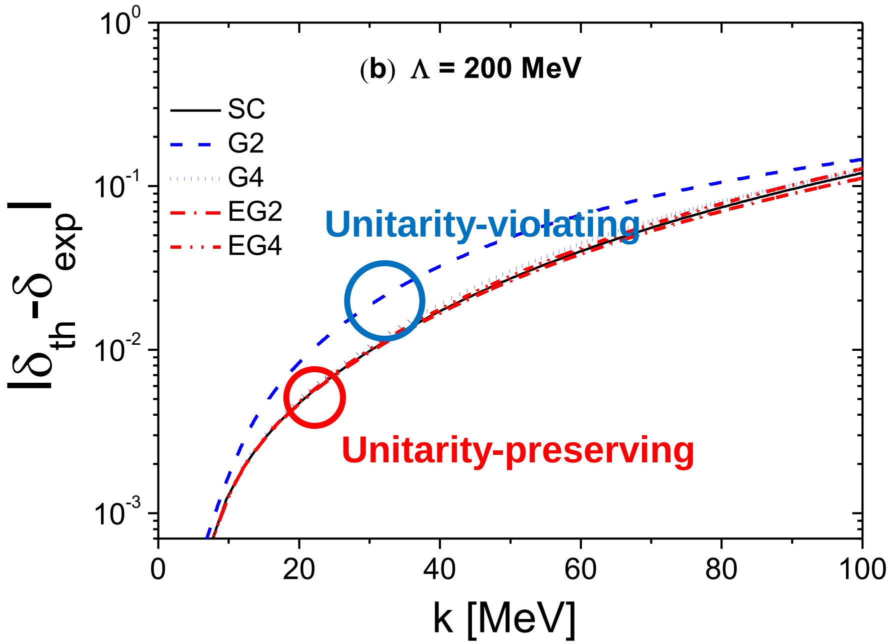
(G: gaussian form, EG: energy-dependent gaussian form, SC: sharp cutoff)

3.1 Role of unitarity in EFT descriptions

Results: EFT with only nucleons

- The np $^1S_0$ phase shifts:
  - Comparison of δ_{th} with δ_{exp} , $|\delta_{th} - \delta_{exp}|$



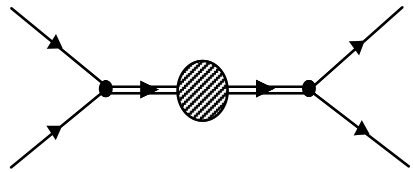


⇒ **unitarity-preserving regulators(EG2, EG4, SC): better agreement**

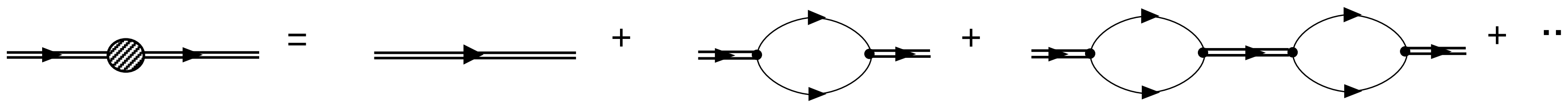
3.1 Role of unitarity in EFT descriptions
Dibaryon field

- $\mathcal{L} = N^\dagger\left(i\partial_t + \frac{\nabla^2}{2M}\right)N - D^\dagger\left(i\partial_t + \frac{\nabla^2}{4M} - \Delta m\right)D - \left(\frac{g}{2}D^\dagger NN + h.c\right) + \cdots.$ [1)]

  - Scattering amplitude



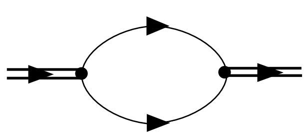
  - Dressed di-baryon propagator



$$iS_D(P) = \frac{-i}{P^0 - \frac{\vec{P}^2}{4M} - \Delta m + \Pi_D(P) + i\epsilon}$$

with $P^\mu = (P^0, \vec{P}),\ E = P^0 - \frac{P^2}{4M}$

  - Self-energy



$$\Pi_{\mathrm{D}}(P) = \int \frac{d^3\vec{l}}{(2\pi)^3}\boldsymbol{R_\Lambda(l)}\frac{g^2}{P^0 - \frac{l^2}{M} - \frac{k^2}{4M} + i\epsilon} = -\frac{Mg^2}{4\pi}\left(-J_\Lambda(k) + \boldsymbol{ik\,R_\Lambda(k)}\right)$$

- Results $\boldsymbol{k\cot\delta} = -\frac{4\pi}{M}\left(-\frac{k^2}{Mg^2} + \frac{\Delta m}{g^2}\right) + J_\Lambda(k) + \underline{ik(1 - R_\Lambda(k))}$

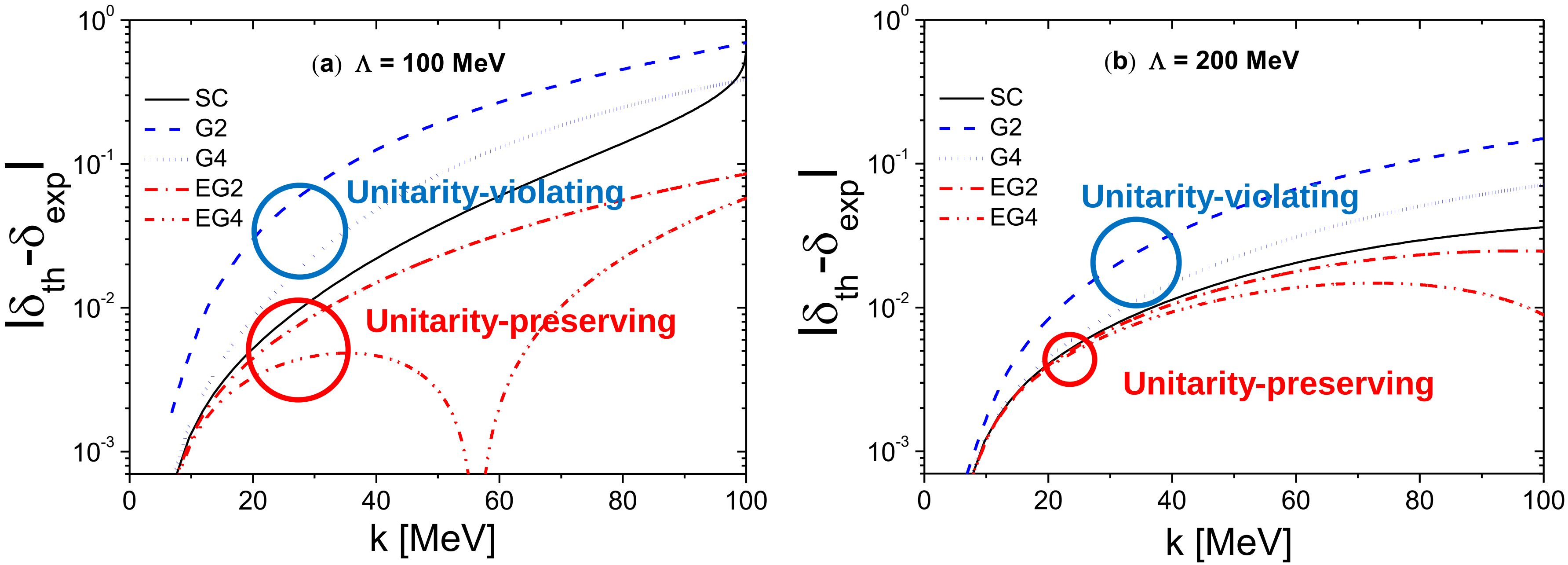
$$J_\Lambda(k) \equiv \frac{2}{\pi}P\int_0^\infty dl\, R(l)\frac{l^2}{k^2 - l^2}$$

[1] D.B. Kaplan et al., Nucl. Phys. B **478**, 629 (1996).

3.1 Role of unitarity in EFT descriptions

Results: Dibaryon field

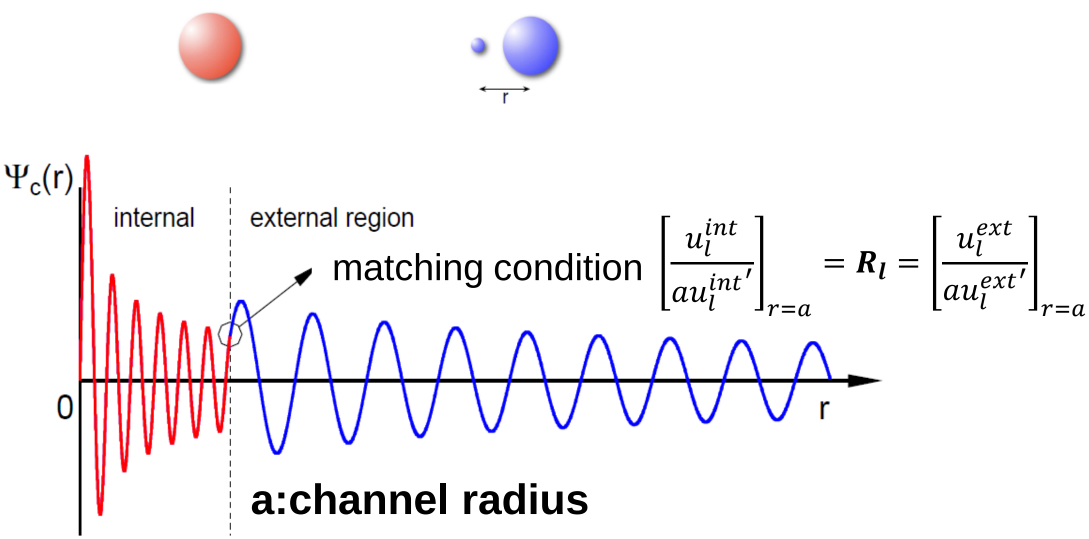
- The np $^1S_0$ phase shifts:
  - Comparison of δ_{th} with δ_{exp} , $|\delta_{th} - \delta_{exp}|$



⇒ **unitarity-preserving regulators(EG2, EG4, SC): better agreement**

3.2 Relation between EFT and R-matrix
R-matrix

- R-matrix[1,2,3]
  - Main idea[2]: to divide the space into 2 regions (channel radius a)
    - Internal ($r \le a$): Nuclear + Coulomb interactions
    - External ($r > a$): Only Coulomb



R-matrix parameters have dependence on channel radius.

- R-function

$$R_l = \sum_\lambda \frac{\gamma_\lambda \gamma_\lambda}{E_\lambda - E}$$

E_λ: resonance energy

$$\boldsymbol{\gamma_\lambda} = \sqrt{\frac{\hbar^2}{2\mu a}}\, \boldsymbol{u_\lambda(a)}$$

reduced-width amplitude

- R-matrix

$$\boldsymbol{R_{cc'}} = \sum_\lambda \frac{\boldsymbol{\gamma_{\lambda c}\gamma_{\lambda c'}}}{\boldsymbol{E_\lambda - E}}$$

[1] E. P. Wigner and L. Eisenbud, Phys. Rev. **72**, 29 (1947). [2] P.L. Kapur and R. Peierls, Proc. Roy. Soc. A **166,** 277-295 (1938).
[3] A.M. Lane and R. G. Thomas, Rev. Mod. Phys. **30**, 257-353 (1958).

3.2 Relation between EFT and R-matrix

R-matrix, ERE and EFT

- T.Teichmann[1)] showed the possibility to find the connection between R-matrix and ERE.
- The relation between the ERE and the R-matrix theory by G.M.Hale[2)].
  - np elastic scattering in the $^1S_0$ channel
  - channel radius dependence of the R-matrix parameters $\Gamma,\ E_r$
  - effective range parameters in ERE $a_0,\ r_0$

R-matrix $\Longleftrightarrow$ Effective Field Theory (EFT)

Relation between $\boldsymbol{a_p}$ from the R-matrix and $\boldsymbol{\Lambda_p}$ from the EFT

- $\boldsymbol{a_p}$: lower bound of the channel radius
- $\boldsymbol{\Lambda_p}$: upper bound of the momentum cutoff

[1] T.Teichmann, Phys. Rev. 83, 141 (1951) [2] G.M. Hale, Phys.Rev.C **89**, 014623 (2014)

3.2 Relation between EFT and R-matrix

Scattering amplitude in R-matrix

- General expression for single-level and single-channel cases, R-matrix/R-function

$$R_l(E;a) = \frac{{\gamma_{l\lambda}}^2(a)}{E_\lambda(a) - E}$$

where a is the channel radius,
$\gamma_{l\lambda}$ is the reduced-width amplitudes

- The scattering matrix U_l

$$U_l = e^{2i\delta_l(E)} = \left[\frac{w_l^{(-)}(E,r)}{w_l^{(+)}(E,r)} \frac{1-(L_l - B_l)^* R_l(E;a)}{1-(L_l - B_l) R_l(E;a)}\right]_{r=a}$$

where $w_l^{(+)}(E,r)$ and $w_l^{(-)}(E,r)$ are the outgoing and ingoing Coulomb waves,
$L_l = \left[r\frac{w_l^{(+)}(E,r)/dr}{w_l^{(+)}(E,r)}\right]_{r=a}$, $L_l^* = \left[r\frac{w_l^{(-)}(E,r)/dr}{w_l^{(-)}(E,r)}\right]_{r=a}$, B_l is the arbitrary boundary constant,
$k = \sqrt{2\mu E}/\hbar$ is the wave number in the center of mass,
μ the reduced mass of the scattering pair, and E_λ the energy eigenvalues.

- let us define $g_R^2 \equiv a\gamma_{l\lambda}^2$

- The scattering amplitude

$$k\cot\delta_l(E) = \frac{\boldsymbol{E_\lambda} - \frac{\hbar^2 k^2}{2\mu} + k g_R^2 \tan ka + g_R^2 B_l \frac{1}{a}}{\boldsymbol{g_R^2} - \left(E_\lambda - \frac{\hbar^2 k^2}{2\mu}\right)\frac{1}{k}\tan ka - g_R^2 B_l \frac{1}{ak}\tan ka}$$

$$= \frac{E_\lambda}{g_R^2 - aE_\lambda} + \frac{3ag_R^4 - 3g_R^2\left(a^2 E_\lambda + \frac{\hbar^2}{2\mu}\right) - a^3 E_\lambda}{3(g_R^2 - aE_\lambda)^2} k^2 + \cdots$$

R-matrix and EFT

- A spin-singlet $^1S_0$ channel of np scattering
- Relation between the EFT and R-matrix
  - Effective Field Theory (EFT)

$$k\cot\delta = -\frac{4\pi}{M}\left(-\frac{k^2}{Mg^2}+\frac{\boldsymbol{\Delta m}}{\boldsymbol{g^2}}\right)+\boldsymbol{J_\Lambda(k)}+ik(1-R_\Lambda(k))$$

  - R-matrix

$$k\cot\delta = \frac{E_\lambda-\frac{\hbar^2k^2}{2\mu}+kg_R^2\tan ka}{g_R^2-\left(E_\lambda-\frac{\hbar^2k^2}{2\mu}\right)\frac{1}{k}\tan ka} = \frac{\boldsymbol{E_\lambda}}{\boldsymbol{g_R^2-aE_\lambda}}+\frac{3ag_R^4-3g_R^2\left(a^2E_\lambda+\frac{\hbar^2}{2\mu}\right)-a^3E_\lambda}{3(g_R^2-aE_\lambda)^2}k^2+\cdots$$

- Relation between the EFT and R-matrix

$$\frac{1}{\frac{4\pi}{M}\frac{\Delta\mathrm{m}}{g^2}-\boldsymbol{J_\Lambda(0)}}=\boldsymbol{a}-\frac{g_R^2}{E_\lambda}$$

where $J_\Lambda(0)\sim\Lambda$

$$\boldsymbol{\Lambda\sim\frac{1}{a}}$$

3.2 Relation between EFT and R-matrix

Critical values of channel radius in R-matrix

- A spin-singlet ${}^1S_0$ channel of np scattering
  - R-matrix
    - $k\cot\delta_l(E) = \frac{E_\lambda}{g_R^2 - aE_\lambda} + \frac{3ag_R^4 - 3g_R^2\left(a^2E_\lambda + \frac{\hbar^2}{2\mu}\right) - a^3E_\lambda}{3\left(g_R^2 - aE_\lambda\right)^2} k^2 + \cdots$
    - $a_0 = (-23.740 \pm 0.020)\ \text{fm},\ r_0 = (2.77 \pm 0.05)\ \text{fm}$
  - ERE
    - $k\cot\delta = -\frac{1}{\boldsymbol{a_0}} + \frac{1}{2}\boldsymbol{r_0}k^2 + \cdots$
- Relation between R-matrix parameters and ERE parameters[1)]

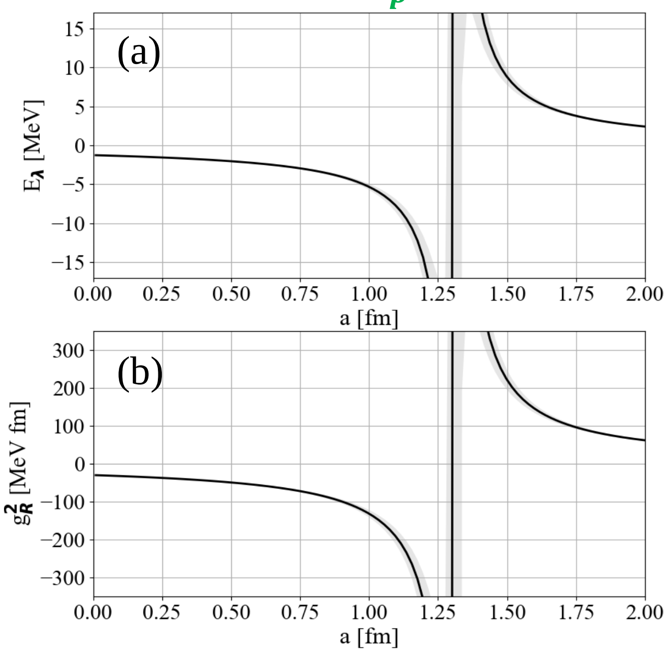
$$E_\lambda(a) = \frac{\hbar^2}{2\mu}(a_0 - a)\left[\frac{r_0 a_0^2}{2} - \frac{a^3}{3} - aa_0(a_0 - a)\right]^{-1}$$

$$g_R^2(a) = -\frac{\hbar^2}{2\mu}(a_0 - a)^2\left[\frac{r_0 a_0^2}{2} - \frac{a^3}{3} - aa_0(a_0 - a)\right]^{-1}$$

at least, one pole

$$\boldsymbol{a_p(a_0, r_0) = a_0 - a_0\left(1 - \frac{3r_0}{2a_0}\right)^{\frac{1}{3}}}$$

$\boldsymbol{a_p} \sim \mathbf{1.3}\ \text{fm}$



[1] G.M. Hale, Phys.Rev.C **89**, 014623 (2014)

3.2 Relation between EFT and R-matrix

Critical values of momentum cutoff in EFT

- For ${}^1S_0$ channel of np scattering, we determine the values of the LECs (Δm, g^2).
  - experimental values
    $a_0 = (-23.740 \pm 0.020)$ fm, $r_0 = (2.77 \pm 0.05)$ fm
  - Effective Field Theory (EFT)

$$k\cot\delta = -\frac{4\pi}{M}\left(-\frac{k^2}{M\boldsymbol{g^2}} + \frac{\boldsymbol{\Delta m}}{g^2}\right) + J_\Lambda(k) + ik(1 - R_\Lambda(k))$$

$$\frac{M\boldsymbol{g^2}}{4\pi} = \frac{2}{M}\left(r_0 - \underline{\frac{\partial}{\partial k^2}J^\Lambda(k)\bigg|_{k=0}}\right)^{-1}$$

$$\boldsymbol{\Delta m} = \frac{Mg^2}{4\pi}\left(\frac{1}{a_0} + J^\Lambda(0)\right)$$

where $J_\Lambda(k) \equiv \frac{2}{\pi} P \int_0^\infty dl\, R(l) \frac{l^2}{k^2 - l^2}$

$$\boldsymbol{\Lambda_p(r_0; R^\Lambda) = \frac{\Lambda}{r_0}\frac{\partial}{\partial k^2}J_\Lambda(k^2)\bigg|_{k^2=0}}$$

  - ERE

$$k\cot\delta = -\frac{1}{\boldsymbol{a_0}} + \frac{1}{2}\boldsymbol{r_0}k^2 + \cdots$$

Relation between two critical values

- A spin-singlet $^1S_0$ channel of np scattering
  - R-matrix

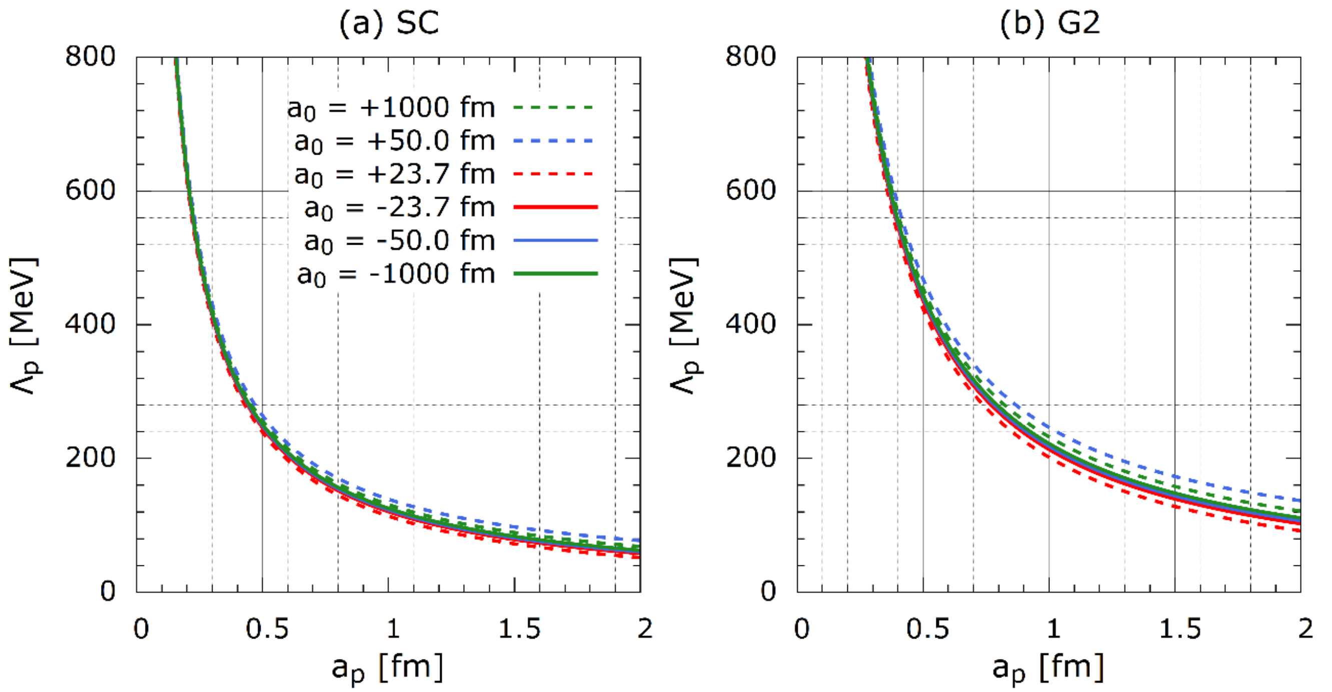
$$a_p(a_0, r_0) = a_0 - a_0\left(1 - \frac{3r_0}{2a_0}\right)^{\frac{1}{3}}$$

  - EFT

$$\Lambda_p(a_0, r_0; R^\Lambda) = \frac{\Lambda}{r_0}\frac{\partial}{\partial k^2}J_\Lambda(k^2)\bigg|_{k^2=0}$$

- The behavior of the Λ_p and a_p in accordance with varying a_0, r_0

$$\boldsymbol{\Lambda_p \sim \frac{1}{a_p}}$$



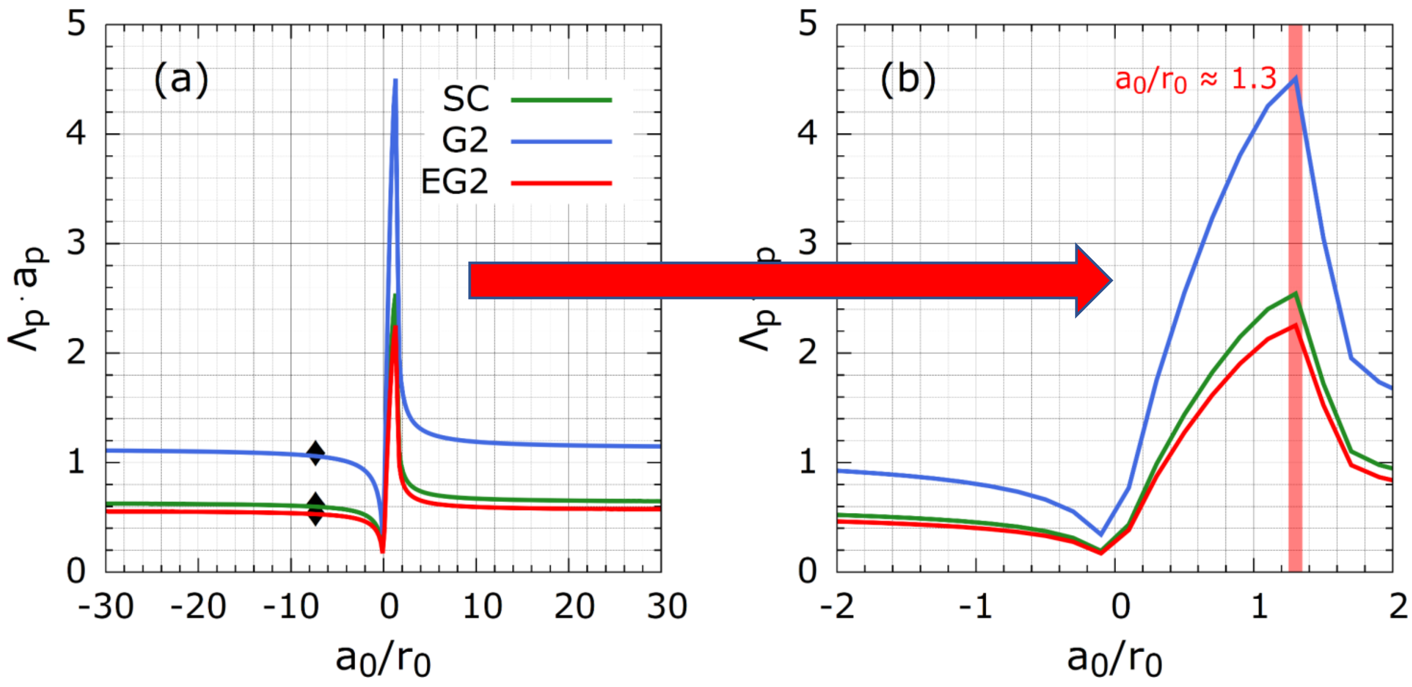
3.2 Relation between EFT and R-matrix

Relation between two critical values

- A spin-singlet ${}^1S_0$ channel of np scattering

$$P(a_0, r_0; R^\Lambda) = \mathrm{a_p}\Lambda_p = C\left(\frac{a_0}{r_0} - \frac{a_0}{r_0}\left(1 - \frac{3r_0}{2a_0}\right)^{\frac{1}{3}}\right) \qquad P(a_0, r_0; R^\Lambda) = a_{ch}^{min}\Lambda_p \propto \frac{a_0}{r_0}.$$

- The behavior of the $P(a_0, r_0; R^\Lambda)$ in accordance with varying a_0/r_0



3.2 Relation between EFT and R-matrix

Summary

- The R-matrix, which is one of the methods to describe the resonance in nuclei, determines the resonance parameters (energy eigenvalues of levels and reduced-width) based on experimental data.

- The resonance parameters show dependence on the channel radius, which is the boundary to divide the configuration space into two region: internal and external regions.

- In EFT, LECs have a dependence on momentum cutoff Λ.

- For the first, we found the relation between two critical values (a_p, Λ_p) of the channel radius a in R-matrix and momentum cutoff Λ in EFT.

- As a results, we can see the possibility to give constraint of these two arbitrary values.

- Future work
  - Explore to other reactions such as elastic p+^{12}C scattering

Thank you for your attention